VIII. ВЕКТОРНЫЙ АНАЛИЗ

Расчетные задания

Задача 1. Найти производную скалярного поля $u(x, y, z)$ в точке M по направлению нормали к поверхности S, образующей острый угол с положительным направлением оси Oz.

1.1. $u = 4\ln(3 + x^2) - 8xyz, \quad S:\ x^2 - 2y^2 + 2z^2 = 1, \quad M(1,\ 1,\ 1).$

1.2. $u = x\sqrt{y} + y\sqrt{z}, \quad S:\ 4\mathrm{z} + 2x^2 - y^2 = 0, \quad M(2,\ 4,\ 4).$

1.3. $u = -2\ln(x^2 - 5) - 4xyz, \quad S:\ x^2 + 2y^2 - 2z^2 = 1, \quad M(1,\ 1,\ 1).$

1.4. $u = \dfrac{1}{4}x^2 y - \sqrt{x^2 + 5z^2}, \quad S:\ z^2 = x^2 + 4y^2 - 4, \quad M\left(-2,\ \dfrac{1}{2},\ 1\right).$

1.5. $u = xz^2 - \sqrt{x^3 y}, \quad S:\ x^2 - y^2 - 3z + 12 = 0, \quad M(2,\ 2,\ 4).$

1.6. $u = x\sqrt{y} - yz^2, \quad S:\ x^2 + y^2 = 4z, \quad M(2,\ 1,\ \text{-}1).$

1.7. $u = 7\ln(1/13 + x^2) - 4xyz, \quad S:\ 7x^2 - 4y^2 + 4z^2 = 7, \quad M(1,\ 1,\ 1).$

1.8. $u = \operatorname{arctg}(y/x) - 8xyz, \quad S:\ x^2 + y^2 - 2z^2 = 10, \quad M(2,\ 2,\ \text{-}1).$

1.9. $u = \ln(1 + x^2) - xy\sqrt{z}, \quad S:\ 4x^2 - y^2 + z^2 = 16, \quad M(1,\ \text{-}2,\ 4).$

1.10. $u = \sqrt{x^2 + y^2} - z, \quad S:\ x^2 + y^2 = 24z, \quad M(3,\ 4,\ 1).$

1.11. $u = x\sqrt{y} - (z + y)\sqrt{x}, \quad S:\ x^2 - y^2 + z^2 = 4, \quad M(1,\ 1,\ \text{-}2).$

1.12. $u = \sqrt{xy} - \sqrt{4 - z^2}, \quad S:\ z = x^2 - y^2, \quad M(1,\ 1,\ 0).$

1.13. $u = (x^2 + y^2 + z^2)^{3/2}, \quad S:\ 2x^2 - y^2 + z^2 - 1 = 0, \quad M(0,\ \text{-}3,\ 4).$

1.14. $u = \ln(1 + x^2 + y^2) - \sqrt{x^2 + z^2}, \quad S:\ x^2 - 6x + 9y^2 + z^2 = 4z + 4, \quad M(3,\ 0,\ \text{-}4).$

Найти производную скалярного поля $u(x, y, z)$ в точке M по направлению вектора $\mathbf{l}$.

$u=\left(x^2+y^2+z^2\right)^{3/2}$,

1.15. $\mathbf{l}=\mathbf{i}-\mathbf{j}+\mathbf{k}$,

$M(1,\ 1,\ 1)$.

$u=x+\ln\left(z^2+y^2\right)$,

1.16. $\mathbf{l}=-2\mathbf{i}+\mathbf{j}-\mathbf{k}$,

$M(2,\ 1,\ 1)$.

$u=x^2y-\sqrt{xy+z^2}$,

1.17. $\mathbf{l}=2\mathbf{j}-2\mathbf{k}$,

$M(1,\ 5,\ -2)$.

$u=y\ln\left(1+x^2\right)-\operatorname{arctg} z$,

1.18. $\mathbf{l}=2\mathbf{i}-3\mathbf{j}-2\mathbf{k}$,

$M(0,\ 1,\ 1)$.

$u=x(\ln y-\operatorname{arctg} z)$,

1.19. $\mathbf{l}=8\mathbf{i}+4\mathbf{j}+8\mathbf{k}$,

$M(-2,\ 1,\ -1)$.

$u=\ln\left(3-x^2\right)+xy^2z$,

1.20. $\mathbf{l}=-\mathbf{i}+2\mathbf{j}-2\mathbf{k}$,

$M(1,\ 3,\ 2)$.

$u=\sin(x+2y)+\sqrt{xyz}$,

1.21. $\mathbf{l}=4\mathbf{i}+3\mathbf{j}$,

$M(\pi/2,\ 3\pi/2,\ 3)$.

$u=x^2y^2z-\ln(z-1)$,

1.22. $\mathbf{l}=5\mathbf{i}-6\mathbf{j}+2\sqrt{5}\mathbf{k}$,

$M(1,\ 1,\ 2)$.

$u=x^3+\sqrt{y^2+z^2}$,

1.23. $\mathbf{l}=\mathbf{j}-\mathbf{k}$,

$M(1,\ -3,\ 4)$.

$u=\dfrac{\sqrt{x}}{y}-\dfrac{yz}{x+\sqrt{y}}$,

1.24. $\mathbf{l}=2\mathbf{i}+\mathbf{k}$,

$M(4,\ 1,\ -2)$.

$u=\sqrt{xy}+\sqrt{9-z^2}$,

1.25. $\mathbf{l}=-2\mathbf{i}+2\mathbf{j}-\mathbf{k}$,

$M(1,\ 1,\ 0)$.

$u=2\sqrt{x+y}+y\operatorname{arctg} z$,

1.26. $\mathbf{l}=4\mathbf{i}-3\mathbf{k}$,

$M(3,\ 2,\ -1)$.

$u=z^2+2\operatorname{arctg}(x-y)$,

1.27. $\mathbf{l}=\mathbf{i}+2\mathbf{j}-2\mathbf{k}$,

$M(1,\ 2,\ -1)$.

$u=\ln\left(x^2+y^2\right)+xyz$,

1.28. $\mathbf{l}=\mathbf{i}-\mathbf{j}+5\mathbf{k}$,

$M(1,\ -1,\ 2)$.

$u = xy - \frac{x}{z}$,

1.29. $\mathbf{l} = 5\mathbf{i} + \mathbf{j} - \mathbf{k}$,

$M(-4,\ 3,\ -1)$.

$u = \ln\left(x + \sqrt{y^2 + z^2}\right)$,

1.30. $\mathbf{l} = -2\mathbf{i} - \mathbf{j} + \mathbf{k}$,

$M(1,\ -3,\ 4)$.

$u = x^2 - \operatorname{arctg}(y + z)$,

1.31. $\mathbf{l} = 3\mathbf{j} - 4\mathbf{k}$,

$M(2,\ 1,\ 1)$.

Задача 2. Найти угол между градиентами скалярных полей $u(x,\ y\ ,z)$ и $\upsilon(x,\ y,\ z)$ в точке M .

2.1. $\upsilon = \frac{x^3}{2} + 6y^3 + 3\sqrt{6}z^3, \quad u = \frac{yz^2}{x^2}, \quad M\left(\sqrt{2},\ \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}}\right)$.

2.2. $\upsilon = \frac{4\sqrt{6}}{x} - \frac{\sqrt{6}}{9y} + \frac{3}{z}, \quad u = x^2yz^3, \quad M\left(2,\ \frac{1}{3},\ \sqrt{\frac{3}{2}}\right)$.

2.3. $\upsilon = 9\sqrt{2}x^3 - \frac{y^3}{2\sqrt{2}} - \frac{4z^3}{\sqrt{3}}, \quad u = \frac{z^3}{xy^2}, \quad M\left(\frac{1}{3},\ 2,\ \sqrt{\frac{3}{2}}\right)$.

2.4. $\upsilon = \frac{3}{x} + \frac{4}{y} - \frac{1}{\sqrt{6}z}, \quad u = \frac{z}{x^3y^2}, \quad M\left(1,\ 2,\ \frac{1}{\sqrt{6}}\right)$.

2.5. $\upsilon = \frac{x^3}{2} + 6y^3 + 3\sqrt{6}z^3, \quad u = \frac{x^2}{yz^2}, \quad M\left(\sqrt{2},\ \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}}\right)$.

2.6. $\upsilon = 3\sqrt{2}x^2 - \frac{y^2}{\sqrt{2}} + 3\sqrt{2}z^3, \quad u = \frac{z^2}{xy^2}, \quad M\left(\frac{1}{3},\ 2,\ \sqrt{\frac{2}{3}}\right)$.

2.7. $\upsilon = 6\sqrt{6}x^3 - 6\sqrt{6}y^3 + 2z^3, \quad u = \frac{xz^2}{y}, \quad M\left(\frac{1}{\sqrt{6}},\ \frac{1}{\sqrt{6}},\ 1\right)$.

2.8. $\upsilon = \frac{\sqrt{6}}{2x} - \frac{\sqrt{6}}{2y} + \frac{2}{3z}, \quad u = \frac{yz^2}{x}, \quad M\left(\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}}\right)$.

2.9. $\upsilon = 3\sqrt{2}x^2 - \dfrac{y^2}{\sqrt{2}} - 3\sqrt{2}z^2, \quad u = \dfrac{xy^2}{z^2}, \quad M\left(\dfrac{1}{3},\ 2,\ \sqrt{\dfrac{2}{3}}\right).$

2.10. $\upsilon = \dfrac{3}{x} + \dfrac{4}{y} - \dfrac{1}{\sqrt{6}z}, \quad u = \dfrac{x^3y^2}{z}, \quad M\left(1,\ 2,\ \dfrac{1}{\sqrt{6}}\right).$

2.11. $\upsilon = -\dfrac{4\sqrt{2}}{x} + \dfrac{\sqrt{2}}{9y} + \dfrac{1}{\sqrt{3}z}, \quad u = \dfrac{1}{x^2yz}, \quad M\left(2,\ \dfrac{1}{3},\ \dfrac{1}{\sqrt{6}}\right).$

2.12. $\upsilon = \dfrac{6}{x} + \dfrac{2}{y} - \dfrac{3\sqrt{3}}{2\sqrt{2}z}, \quad u = \dfrac{x^2}{y^2z^3}, \quad M\left(\sqrt{2},\ \sqrt{2},\ \dfrac{\sqrt{3}}{2}\right).$

2.13. $\upsilon = x^2 + 9y^2 + 6z^2, \quad u = xyz, \quad M\left(1,\ \dfrac{1}{3},\ \dfrac{1}{\sqrt{6}}\right).$

2.14. $\upsilon = \dfrac{2}{x} + \dfrac{3}{2y} - \dfrac{\sqrt{6}}{4z}, \quad u = \dfrac{y^3}{x^2z}, \quad M\left(\sqrt{\dfrac{2}{3}},\ \sqrt{\dfrac{3}{2}},\ \dfrac{1}{2}\right).$

2.15. $\upsilon = \sqrt{2}x^2 - \dfrac{3y^2}{\sqrt{2}} - 6\sqrt{2}z^2, \quad u = xy^2z, \quad M\left(1,\ \dfrac{2}{3},\ \dfrac{1}{\sqrt{6}}\right).$

2.16. $\upsilon = -\dfrac{\sqrt{6}}{2x} + \dfrac{\sqrt{6}}{2y} - \dfrac{2}{3z}, \quad u = \dfrac{x}{yz^2}, \quad M\left(\dfrac{1}{\sqrt{2}},\ \dfrac{1}{\sqrt{2}},\ \dfrac{1}{\sqrt{3}}\right).$

2.17. $\upsilon = \dfrac{6}{x} + \dfrac{2}{y} + \dfrac{3\sqrt{3}}{2\sqrt{2}z}, \quad u = \dfrac{y^2z^3}{x^2}, \quad M\left(\sqrt{2},\ \sqrt{2},\ \dfrac{\sqrt{3}}{2}\right).$

2.18. $\upsilon = \dfrac{1}{\sqrt{2}x} - \dfrac{2\sqrt{2}}{y} - \dfrac{3\sqrt{3}}{2z}, \quad u = \dfrac{y^2z^3}{x}, \quad M\left(\dfrac{1}{\sqrt{2}},\ \sqrt{2},\ \dfrac{\sqrt{3}}{2}\right).$

2.19. $\upsilon = 6\sqrt{6}x^3 - 6\sqrt{6}y^3 + 2z^3, \quad u = \dfrac{y}{xz^2}, \quad M\left(\dfrac{1}{\sqrt{6}},\ \dfrac{1}{\sqrt{6}},\ ?\right).$

2.20. $\upsilon = x^2 - y^2 - 3z^2, \quad u = \dfrac{yz^2}{x}, \quad M\left(\dfrac{1}{\sqrt{2}},\ \dfrac{1}{\sqrt{2}},\ \dfrac{1}{\sqrt{3}}\right).$

2.21. $\upsilon = \dfrac{3x^2}{\sqrt{2}} - \dfrac{y^2}{\sqrt{2}} + \sqrt{2}z^2, \quad u = \dfrac{z^2}{x^2y^2}, \quad M\left(\dfrac{2}{3},\ 2,\ \sqrt{\dfrac{2}{3}}\right).$

2.22. $\upsilon = \frac{x^3}{\sqrt{2}} - \frac{y^3}{\sqrt{2}} - \frac{8z^3}{\sqrt{3}}, \quad u = \frac{x^2}{y^2 z^3}, \quad M\left(\sqrt{2},\ \sqrt{2},\ \frac{\sqrt{3}}{2}\right).$

2.23. $\upsilon = \frac{3}{2}x^2 + 3y^2 - 2z^2, \quad u = x^2 y z^3, \quad M\left(2,\ \frac{1}{3},\ \sqrt{\frac{3}{2}}\right).$

2.24. $\upsilon = 9\sqrt{2}x^3 - \frac{y^3}{2\sqrt{2}} - \frac{4z^3}{\sqrt{3}}, \quad u = \frac{xy^2}{z^3}, \quad M\left(\frac{1}{3},\ 2,\ \sqrt{\frac{3}{2}}\right).$

2.25. $\upsilon = \sqrt{2}x^2 - \frac{3y^2}{\sqrt{2}} - 6\sqrt{2}z^2, \quad u = \frac{1}{xy^2 z}, \quad M\left(1,\ \frac{2}{3},\ \frac{1}{\sqrt{6}}\right).$

2.26. $\upsilon = x^2 + 9y^2 + 6z^2, \quad u = \frac{1}{xyz}, \quad M\left(1,\ \frac{1}{3},\ \frac{1}{\sqrt{6}}\right).$

2.27. $\upsilon = \frac{1}{\sqrt{2}x} - \frac{2\sqrt{2}}{y} - \frac{3\sqrt{3}}{2z}, \quad u = \frac{x}{y^2 z^3}, \quad M\left(\frac{1}{\sqrt{2}},\ \sqrt{2},\ \frac{\sqrt{3}}{2}\right).$

2.28. $\upsilon = -\frac{4\sqrt{2}}{x} + \frac{\sqrt{2}}{9y} + \frac{1}{\sqrt{3}z}, \quad u = x^2 yz, \quad M\left(2,\ \frac{1}{3},\ \frac{1}{\sqrt{6}}\right).$

2.29. $\upsilon = \frac{x^3}{\sqrt{2}} - \frac{y^3}{\sqrt{2}} - \frac{8z^3}{\sqrt{3}}, \quad u = \frac{y^2 z^3}{x^2}, \quad M\left(\sqrt{2},\ \sqrt{2},\ \frac{\sqrt{3}}{2}\right).$

2.30. $\upsilon = -\frac{3x^3}{\sqrt{2}} + \frac{2\sqrt{2}y^3}{3} + 8\sqrt{3}z^3, \quad u = \frac{x^2 z}{x^2}, \quad M\left(\sqrt{\frac{2}{3}},\ \sqrt{\frac{3}{2}},\ \frac{1}{2}\right).$

2.31. $\upsilon = x^2 - y^2 - 3z^2, \quad u = \frac{x}{yz^2}, \quad M\left(\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{3}}\right).$

Задача 3. Найти векторные линии в векторном поле $\mathbf{a}$.

3.1. $\mathbf{a} = 4y\mathbf{i} - 9x\mathbf{j}.$

3.2. $\mathbf{a} = 2y\mathbf{i} + 3x\mathbf{j}.$

3.3. $\mathbf{a} = 2x\mathbf{i} + 4y\mathbf{j}.$

3.4. $\mathbf{a} = x\mathbf{i} + 3y\mathbf{j}.$

3.5. $\mathbf{a} = x\mathbf{i} + 4y\mathbf{j}.$

3.6. $\mathbf{a} = 3x\mathbf{i} + 6z\mathbf{k}.$

3.7. $\mathbf{a} = 4z\mathbf{i} - 9x\mathbf{k}.$

3.8. $\mathbf{a} = 2z\mathbf{i} + 3x\mathbf{k}.$

3.9. $\mathbf{a} = 4y\mathbf{j} + 8z\mathbf{k}.$

3.10. $\mathbf{a} = y\mathbf{j} + 3z\mathbf{k}.$

3.11. $\mathbf{a} = 2x\mathbf{i} + 8z\mathbf{k}$.

3.12. $\mathbf{a} = x\mathbf{i} + 3z\mathbf{k}$.

3.13. $\mathbf{a} = 4z\mathbf{j} - 9y\mathbf{k}$.

3.14. $\mathbf{a} = 2z\mathbf{j} + 3y\mathbf{k}$.

3.15. $\mathbf{a} = 5x\mathbf{i} + 10y\mathbf{j}$.

3.16. $\mathbf{a} = 2x\mathbf{i} + 6y\mathbf{j}$.

3.17. $\mathbf{a} = y\mathbf{j} + 4z\mathbf{k}$.

3.18. $\mathbf{a} = x\mathbf{i} + y\mathbf{j}$.

3.19. $\mathbf{a} = 9y\mathbf{i} - 4x\mathbf{j}$.

3.20. $\mathbf{a} = 5y\mathbf{i} + 7x\mathbf{j}$.

3.21. $\mathbf{a} = 6x\mathbf{i} + 12z\mathbf{k}$.

3.22. $\mathbf{a} = 2y\mathbf{j} + 6z\mathbf{k}$.

3.23. $\mathbf{a} = 4x\mathbf{i} + y\mathbf{j}$.

3.24. $\mathbf{a} = 9z\mathbf{i} - 4x\mathbf{k}$.

3.25. $\mathbf{a} = x\mathbf{i} + z\mathbf{k}$.

3.26. $\mathbf{a} = 5z\mathbf{i} + 7x\mathbf{k}$.

3.27. $\mathbf{a} = 7y\mathbf{j} + 14z\mathbf{k}$.

3.28. $\mathbf{a} = 2x\mathbf{i} + 6z\mathbf{k}$.

3.29. $\mathbf{a} = 4x\mathbf{i} + z\mathbf{k}$.

3.30. $\mathbf{a} = 5z\mathbf{j} + 7y\mathbf{k}$.

3.31. $\mathbf{a} = 9z\mathbf{j} - 4y\mathbf{k}$.

Задача 4. Найти поток векторного поля $\mathbf{a}$ через часть поверхности S, вырезаемую плоскостями P_1, P_2 (нормаль внешняя к замкнутой поверхности, образуемой данными поверхностями).

4.1. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.
$S:\ x^2 + y^2 = 1$,
$P_1:\ z = 0,\ P_2:\ z = 2$.

4.2. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} - z\mathbf{k}$.
$S:\ x^2 + y^2 = 1$,
$P_1:\ z = 0,\ P_2:\ z = 4$.

4.3. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + 2z\mathbf{k}$.
$S:\ x^2 + y^2 = 1$,
$P_1:\ z = 0,\ P_2:\ z = 3$.

4.4. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z^3\mathbf{k}$.
$S:\ x^2 + y^2 = 1$,
$P_1:\ z = 0,\ P_2:\ z = 1$.

4.5. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + xyz\mathbf{k}$.
$S:\ x^2 + y^2 = 1$,
$P_1:\ z = 0,\ P_2:\ z = 5$.

4.6. $\mathbf{a} = (x - y)\mathbf{i} + (x + y)\mathbf{j} + z^2\mathbf{k}$.
$S:\ x^2 + y^2 = 1$,
$P_1:\ z = 0,\ P_2:\ z = 2$.

4.7. $\mathbf{a} = (x + y)\mathbf{i} - (x - y)\mathbf{j} + xyz\mathbf{k}$.
$S:\ x^2 + y^2 = 1$,
$P_1:\ z = 0,\ P_2:\ z = 4$.

4.8. $\mathbf{a} = (x^3 + xy^2)\mathbf{i} + (y^3 + x^2y)\mathbf{j} + z^2\mathbf{k}$.
$S:\ x^2 + y^2 = 1$,
$P_1:\ z = 0,\ P_2:\ z = 3$.

$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + \sin z\mathbf{k}.$

4.9. $S: x^2 + y^2 = 1,$

$P_1: z = 0,\ P_2: z = 5.$

$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + \mathbf{k}.$

4.10. $S: x^2 + y^2 = 1,$

$P_1: z = 0,\ P_2: z = 2.$

Найти поток векторного поля $\mathbf{a}$ через часть поверхности S, вырезаемую плоскостью P (нормаль внешняя к замкнутой поверхности, образуемой данными поверхностями).

4.11. $\mathbf{a} = \left(x + xy^2\right)\mathbf{i} + \left(y - yx^2\right)\mathbf{j} + (z-3)\mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 1.$

4.12. $\mathbf{a} = y\mathbf{i} - x\mathbf{j} + \mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 4.$

4.13. $\mathbf{a} = xy\mathbf{i} - x^2\mathbf{j} + 3\mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 1.$

4.14. $\mathbf{a} = xz\mathbf{i} + yz\mathbf{j} + \left(z^2 - 1\right)\mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 4.$

4.15. $\mathbf{a} = y^2x\mathbf{i} - yx^2\mathbf{j} + \mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 5.$

4.16. $\mathbf{a} = (xz + y)\mathbf{i} + (yz - x)\mathbf{j} + \left(z^2 - 2\right)\mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 3.$

4.17. $\mathbf{a} = xyz\mathbf{i} - x^2z\mathbf{j} + 3\mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 2.$

4.18. $\mathbf{a} = (x + xy)\mathbf{i} + \left(y - x^2\right)\mathbf{j} + (z-1)\mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 3.$

4.19. $\mathbf{a} = (x + y)\mathbf{i} + (y - x)\mathbf{j} + (z-2)\mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 2.$

4.20. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + (z-2)\mathbf{k},\quad S: x^2 + y^2 = z^2\ (z \geq 0),\quad P: z = 1.$

4.21. $\mathbf{a} = (x + xz)\mathbf{i} + y\mathbf{j} + \left(z - x^2\right)\mathbf{k},\quad S: x^2 + y^2 + z^2 = 4\ (z \geq 0),\quad P: z = 0.$

4.22. $\mathbf{a} = x\mathbf{i} + \left(y + yz^2\right)\mathbf{j} + \left(z - zy^2\right)\mathbf{k},\quad S: x^2 + y^2 + z^2 = 4,\quad P: z = 0\ (z \geq 0).$

4.23. $\mathbf{a} = (x + z)\mathbf{i} + (y + z)\mathbf{j} + (z - x - y)\mathbf{k},\quad S: x^2 + y^2 + z^2 = 4,\quad P: z = 0\ (z \geq 0).$

4.24. $\mathbf{a} = (x + xy)\mathbf{i} + \left(y - x^2\right)\mathbf{j} + z\mathbf{k},\quad S: x^2 + y^2 + z^2 = 1,\quad P: z = 0\ (z \geq 0).$

4.25. $\mathbf{a} = (x + z)\mathbf{i} + y\mathbf{j} + (z - x)\mathbf{k},\quad S: x^2 + y^2 + z^2 = 1,\quad P: z = 0\ (z \geq 0).$

4.26. $\mathbf{a} = x\mathbf{i} + (y + yz)\mathbf{j} + \left(z - y^2\right)\mathbf{k},\quad S: x^2 + y^2 + z^2 = 1,\quad P: z = 0\ (z \geq 0).$

4.27. $\mathbf{a} = (x - y)\mathbf{i} + (x + y)\mathbf{j} + z\mathbf{k},\quad S: x^2 + y^2 + z^2 = 1,\quad P: z = 0\ (z \geq 0).$

4.28. $\mathbf{a} = \left(x + xz^2\right)\mathbf{i} + y\mathbf{j} + \left(z - zx^2\right)\mathbf{k},\quad S: x^2 + y^2 + z^2 = 9,\quad P: z = 0\ (z \geq 0).$

4.29. $\mathbf{a}=(x+y)\mathbf{i}+(y-x)\mathbf{j}+z\mathbf{k}, \quad S:\ x^2+y^2+z^2=4, \quad P:\ z=0\ (z\geq 0).$

4.30. $\mathbf{a}=(x+xy^2)\mathbf{i}+(y-yx^2)\mathbf{j}+z\mathbf{k}, \quad S:\ x^2+y^2+z^2=9, \quad P:\ z=0\ (z\geq 0).$

4.31. $\mathbf{a}=x\mathbf{i}+(y+z)\mathbf{j}+(z-y)\mathbf{k}, \quad S:\ x^2+y^2+z^2=9, \quad P:\ z=0\ (z\geq 0).$

Задача 5. Найти поток векторного поля $\mathbf{a}$ через часть плоскости P, расположенную в первом октанте (нормаль образует острый угол с осью Oz).

5.1. $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $P:\ x+y+z=1.$

5.2. $\mathbf{a}=y\mathbf{j}+z\mathbf{k}$, $P:\ x+y+z=1.$

5.3. $\mathbf{a}=2x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $P:\ x+y+z=1.$

5.4. $\mathbf{a}=x\mathbf{i}+3y\mathbf{j}+2z\mathbf{k}$, $P:\ x+y+z=1.$

5.5. $\mathbf{a}=2x\mathbf{i}+3y\mathbf{j}$, $P:\ x+y+z=1.$

5.6. $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $P:\ x/2+y+z=1.$

5.7. $\mathbf{a}=x\mathbf{i}+2y\mathbf{j}+z\mathbf{k}$, $P:\ x/2+y+z=1.$

5.8. $\mathbf{a}=y\mathbf{j}+3z\mathbf{k}$, $P:\ x/2+y+z=1.$

5.9. $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $P:\ x+y/2+z/3=1.$

5.10. $\mathbf{a}=2x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $P:\ x+y/2+z/3=1.$

5.11. $\mathbf{a}=3x\mathbf{i}+2z\mathbf{k}$, $P:\ x+y/2+z/2=1.$

5.12. $\mathbf{a}=2x\mathbf{i}+3y\mathbf{j}+z\mathbf{k}$, $P:\ x/3+y+z/2=1.$

5.13. $\mathbf{a}=x\mathbf{i}+3y\mathbf{j}-z\mathbf{k}$, $P:\ x/3+y+z/2=1.$

5.14. $\mathbf{a}=-2x\mathbf{i}+y\mathbf{j}+4z\mathbf{k}$, $P:\ x/3+y+z/2=1.$

5.15. $\mathbf{a}=x\mathbf{i}-y\mathbf{j}+6z\mathbf{k}$, $P:\ x/2+y/3+z=1.$

5.16. $\mathbf{a}=2x\mathbf{i}+5y\mathbf{j}+5z\mathbf{k}$, $P:\ x/2+y/3+z=1.$

5.17. $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $P:\ 2x+y/2+z=1.$

5.18. $\mathbf{a}=2x\mathbf{i}+y\mathbf{j}-2z\mathbf{k}$, $P:\ 2x+y/2+z=1.$

5.19. $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+2z\mathbf{k}$, $P:\ 2x+y/2+z=1.$

5.20. $\mathbf{a}=-x\mathbf{i}+y\mathbf{j}+12z\mathbf{k}$, $P:\ 2x+y/2+z=1.$

5.21. $\mathbf{a}=x\mathbf{i}+3y\mathbf{j}+8z\mathbf{k}$, $P:\ x+2y+z/2=1.$

5.22. $\mathbf{a}=x\mathbf{i}-y\mathbf{j}+6z\mathbf{k}$, $P:\ x+2y+z/2=1.$

5.23. $\mathbf{a} = x\mathbf{i} + 2y\mathbf{j} + 5z\mathbf{k}$
$P:\ x + 2y + \dfrac{z}{2} = 1.$

5.24. $\mathbf{a} = x\mathbf{i} + 4y\mathbf{j} + 5z\mathbf{k}$
$P:\ x + 2y + \dfrac{z}{2} = 1.$

5.25. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
$P:\ 2x + 3y + z = 1.$

5.26. $\mathbf{a} = 2x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
$P:\ 2x + 3y + z = 1.$

5.27. $\mathbf{a} = 2x\mathbf{i} + 3y\mathbf{j} + z\mathbf{k}$
$P:\ 2x + 3y + z = 1.$

5.28. $\mathbf{a} = 2x\mathbf{i} + 3y\mathbf{j} + 4z\mathbf{k}$
$P:\ 2x + 3y + z = 1.$

5.29. $\mathbf{a} = x\mathbf{i} + 9y\mathbf{j} + 8z\mathbf{k}$
$P:\ x + 2y + 3z = 1.$

5.30. $\mathbf{a} = 8x\mathbf{i} + 11y\mathbf{j} + 17z\mathbf{k}$
$P:\ x + 2y + 3z = 1.$

5.31. $\mathbf{a} = -x\mathbf{i} + 2y\mathbf{j} + z\mathbf{k}$
$P:\ x + 2y + 3z = 1.$

Задача 6. Найти поток векторного поля $\mathbf{a}$ через часть плоскости P, расположенную в 1 октанте (нормаль образует острый угол с осью Oz).

6.1. $\mathbf{a} = 7x\mathbf{i} + (5\pi y + 2)\mathbf{j} + 4\pi z\mathbf{k},$
$P:\ x + y/2 + 4z = 1.$

6.2. $\mathbf{a} = 2\pi x\mathbf{i} + (7y + 2)\mathbf{j} + 7\pi z\mathbf{k},$
$P:\ x + y/2 + z/3 = 1.$

6.3. $\mathbf{a} = 9\pi x\mathbf{i} + \mathbf{j} - 3z\mathbf{k},$
$P:\ x/3 + y + z = 1.$

6.4. $\mathbf{a} = (2x + 1)\mathbf{i} - y\mathbf{j} + 3\pi z\mathbf{k},$
$P:\ x/3 + y + 2z = 1.$

6.5. $\mathbf{a} = 7x\mathbf{i} + 9\pi y\mathbf{j} + \mathbf{k},$
$P:\ x + y/3 + z = 1.$

6.6. $\mathbf{a} = \mathbf{i} + 5y\mathbf{j} + 11\pi z\mathbf{k},$
$P:\ x + y + z/3 = 1.$

6.7. $\mathbf{a} = x\mathbf{i} + (\pi z - 1)\mathbf{k},$
$P:\ 2x + y/2 + z/3 = 1.$

6.8. $\mathbf{a} = 5\pi x\mathbf{i} + (9y + 1)\mathbf{j} + 4\pi z\mathbf{k},$
$P:\ x/2 + y/3 + z/2 = 1.$

6.9. $\mathbf{a} = 2\mathbf{i} - y\mathbf{j} + \dfrac{3\pi}{2} z\mathbf{k},$
$P:\ x/3 + y + z/4 = 1.$

6.10. $\mathbf{a} = 9\pi x\mathbf{i} + (5y + 1)\mathbf{j} + 2\pi z\mathbf{k},$
$P:\ 3x + y + z/9 = 1.$

6.11. $\mathbf{a} = 7\pi x\mathbf{i} + 2\pi y\mathbf{j} + (7z + 2)\mathbf{k},$
$P:\ x + y + z/2 = 1.$

6.12. $\mathbf{a} = \pi y\mathbf{i} + (4 - 2z)\mathbf{k},$
$P:\ 2x + y/3 + z/4 = 1.$

6.13. $\mathbf{a} = (3\pi - 1)x\mathbf{i} + (9\pi y + 1)\mathbf{j} + 6\pi z\mathbf{k},$
$P:\ \dfrac{x}{2} + \dfrac{y}{3} + \dfrac{z}{9} = 1.$

6.14. $\mathbf{a} = \pi x\mathbf{i} + \dfrac{\pi}{2} y\mathbf{j} + (4 - 2z)\mathbf{k},$
$P:\ x + \dfrac{y}{3} + \dfrac{z}{4} = 1.$

6.15. $\mathbf{a} = (5y + 3)\mathbf{j} + 11\pi z\mathbf{k},$
$P:\ x + y/3 + 4z = 1.$

6.16. $\mathbf{a} = 9\pi y\mathbf{j} + (7z + 1)\mathbf{k},$
$P:\ x + y + z = 1.$

6.17. $\mathbf{a} = \pi y\mathbf{j} + (1 - 2z)\mathbf{k},$
$P:\ x/4 + y/2 + z = 1.$

6.18. $\mathbf{a} = (27\pi - 1)x\mathbf{i} + (34\pi y + 3)\mathbf{j} + 20\pi z\mathbf{k},$
$P:\ 3x + \dfrac{y}{9} + z = 1.$

6.19. $\mathbf{a} = \pi x\mathbf{i} + 2\mathbf{j} + 2\pi z\mathbf{k},$
$P:\ x/2 + y/3 + z = 1.$

6.20. $\mathbf{a} = 4\pi x\mathbf{i} + 7\pi y\mathbf{j} + (2z + 1)\mathbf{k},$
$P:\ 2x + y/3 + 2z = 1.$

6.21. $\mathbf{a} = 3\pi x\mathbf{i} + 6\pi y\mathbf{j} + 10\mathbf{k},$
$P:\ 2x + y + z/3 = 1.$

6.22. $\mathbf{a} = \pi x\mathbf{i} - 2y\mathbf{j} + \mathbf{k},$
$P:\ 2x + y/6 + z = 1.$

6.23. $\mathbf{a} = (21\pi - 1)x\mathbf{i} + 62\pi y\mathbf{j} + (1 - 2\pi z)\mathbf{k},$
$P:\ 8x + y/2 + z/3 = 1.$

6.24. $\mathbf{a} = \pi x\mathbf{i} + 2\pi y\mathbf{j} + 2\mathbf{k},$
$P:\ x/2 + y/4 + z/3 = 1.$

6.25. $\mathbf{a} = 9\pi x\mathbf{i} + 2\pi y\mathbf{j} + 8\mathbf{k},$
$P:\ 2x + 8y + z/3 = 1.$

6.26. $\mathbf{a} = 7\pi x\mathbf{i} + (4y + 1)\mathbf{j} + 2\pi z\mathbf{k},$
$P:\ x/3 + 2y + z = 1.$

6.27. $\mathbf{a} = 6\pi x\mathbf{i} + 3\pi y\mathbf{j} + 10\mathbf{k},$
$P:\ 2x + y/2 + z/3 = 1.$

6.28. $\mathbf{a}=(\pi-1)x\mathbf{i}+2\pi y\mathbf{j}+(1-\pi z)\mathbf{k},$
$P:\ \frac{x}{4}+\frac{y}{2}+\frac{z}{3}=1.$

6.29. $\mathbf{a}=\frac{\pi}{2}x\mathbf{i}+\pi y\mathbf{j}+(4-2z)\mathbf{k},$
$P:\ x+\frac{y}{3}+\frac{z}{4}=1.$

6.30. $\mathbf{a}=7\pi x\mathbf{i}+4\pi y\mathbf{j}+2(z+1)\mathbf{k},$
$P:\ x/3+y/4+z=1.$

6.31. $\mathbf{a}=5\pi x\mathbf{i}+(1-2y)\mathbf{j}+4\pi z\mathbf{k},$
$P:\ x/2+4y+z/3=1.$

Задача 7. Найти поток векторного поля $\mathbf{a}$ через замкнутую поверхность S (нормаль внешняя).

7.1. $\mathbf{a}=(\mathrm{e}^{z}+2x)\mathbf{i}+\mathrm{e}^{x}\,\mathbf{j}+\mathrm{e}^{y}\,\mathbf{k},\ S:\ x+y+z=1,\ x=0,\ y=0,\ z=0.$

7.2. $\mathbf{a}=(3z^2+x)\mathbf{i}+(\mathrm{e}^{x}-2y)\mathbf{j}+(2z-xy)\mathbf{k},\ S:\ x^2+y^2=z^2,\ \mathrm{z}=1,\ z=4.$

7.3. $\mathbf{a}=(\ln y+7x)\mathbf{i}+(\sin z-2y)\mathbf{j}+(\mathrm{e}^{y}-2z)\mathbf{k},\ S:\ x^2+y^2+z^2=2x+2y+2z-2.$

7.4. $\mathbf{a}=(\cos z+3x)\mathbf{i}+(x-2y)\mathbf{j}+(3z+y^2)\mathbf{k},\ S:\ z^2=36(x^2+y^2),\ z=6.$

7.5. $\mathbf{a}=(\mathrm{e}^{-z}-x)\mathbf{i}+(xz+3y)\mathbf{j}+(z+x^2)\mathbf{k},\ S:\ 2x+y+z=2,\ x=0,\ y=0,\ z=0.$

7.6. $\mathbf{a}=(6x-\cos y)\mathbf{i}-(\mathrm{e}^{x}+z)\mathbf{j}-(2y+3z)\mathbf{k},\ S:\ x^2+y^2=z^2,\ z=1,\ z=2.$

7.7. $\mathbf{a}=(4x-2y^2)\mathbf{i}+(\ln z-4y)\mathbf{j}+(x+3z/4)\mathbf{k},\ S:\ x^2+y^2+z^2=2x+3.$

7.8. $\mathbf{a}=(1+\sqrt{z})\mathbf{i}+(4y-\sqrt{x})\mathbf{j}+xy\mathbf{k},\ S:\ z^2=4(x^2+y^2),\ z=3.$

7.9. $\mathbf{a}=(\sqrt{z}-x)\mathbf{i}+(x-y)\mathbf{j}+(y^2-z)\mathbf{k},\ S:\ 3x-2y+z=6,\ x=0,\ y=0,\ z=0.$

7.10. $\mathbf{a}=(yz+x)\mathbf{i}+(x^2+y)\mathbf{j}+(xy^2+z)\mathbf{k},\ S:\ x^2+y^2+z^2=2z.$

7.11. $\mathbf{a}=(\mathrm{e}^{2y}+x)\mathbf{i}+(x-2y)\mathbf{j}+(y^2+3z)\mathbf{k},\ S:\ x-y+z=1,\ x=0,\ y=0,\ z=0.$

7.12. $\mathbf{a}=\left(\sqrt{z}-2x\right)\mathbf{i}+\left(\mathrm{e}^{x}+3y\right)\mathbf{j}+\sqrt{y+x}\mathbf{k},\ S:\ x^2+y^2=z^2,\ z=2,\ z=5.$

7.13. $\mathbf{a}=\left(\mathrm{e}^{z}+x/4\right)\mathbf{i}+\left(\ln x+y/4\right)\mathbf{j}+\dfrac{z}{4}\mathbf{k},\ S:\ x^2+y^2+z^2=2x+2y-2z-2.$

7.14. $\mathbf{a}=\left(3x-2z\right)\mathbf{i}+\left(z-2y\right)\mathbf{j}+\left(1+2z\right)\mathbf{k},\ S:\ z^2=4\left(x^2+y^2\right),\ z=2.$

7.15. $\mathbf{a}=\left(\mathrm{e}^{y}+2x\right)\mathbf{i}+\left(x-y\right)\mathbf{j}+\left(2z-1\right)\mathbf{k},\ S:\ x+2y+z=2,\ x=0,\ y=0,\ z=0.$

7.16. $\mathbf{a}=\left(x+y^2\right)\mathbf{i}+\left(xz+y\right)\mathbf{j}+\left(\sqrt{x^2+1}+z\right)\mathbf{k},\ S:\ x^2+y^2=z^2,\ z=2,\ z=3.$

7.17. $\mathbf{a}=\left(\mathrm{e}^{y}+2x\right)\mathbf{i}+\left(xz-y\right)\mathbf{j}+\left(1/4\right)\left(\mathrm{e}^{xy}-z\right)\mathbf{k},\ S:\ x^2+y^2+z^2=2y+3.$

7.18. $\mathbf{a}=\left(\sqrt{z}+y\right)\mathbf{i}+3x\mathbf{j}+\left(3z+5x\right)\mathbf{k},\ S:\ z^2=8\left(x^2+y^2\right),\ z=2.$

7.19. $\mathbf{a}=\left(8yz-x\right)\mathbf{i}+\left(x^2-1\right)\mathbf{j}+\left(xy-2z\right)\mathbf{k},\ S:\ 2x+3y-z=6,\ x=0,\ y=0,\ z=0.$

7.20. $\mathbf{a}=\left(y+z^2\right)\mathbf{i}+\left(x^2+3y\right)\mathbf{j}+xy\mathbf{k},\ S:\ x^2+y^2+z^2=2x.$

7.21. $\mathbf{a}=\left(2yz-x\right)\mathbf{i}+\left(xz+2y\right)\mathbf{j}+\left(x^2+z\right)\mathbf{k},\ S:\ y-x+z=1,\ x=0,\ y=0,\ z=0.$

7.22. $\mathbf{a}=\left(\sin z+2x\right)\mathbf{i}+\left(\sin x-3y\right)\mathbf{j}+\left(\sin y+2z\right)\mathbf{k},\ S:\ x^2+y^2=z^2,\ z=3,\ z=6.$

7.23. $\mathbf{a}=\left(\cos z+x/4\right)\mathbf{i}+\left(\mathrm{e}^{x}+y/4\right)\mathbf{j}+\left(\dfrac{z}{4}-1\right)\mathbf{k},\ S:\ x^2+y^2+z^2=2z+3.$

7.24. $\mathbf{a}=\left(\sqrt{z}+1+x\right)\mathbf{i}+\left(2x+y\right)\mathbf{j}+\left(\sin x+z\right)\mathbf{k},\ S:\ \begin{cases} z^2=x^2+y^2, \\ z=1. \end{cases}$

7.25. $\mathbf{a}=\left(5x-6y\right)\mathbf{i}+\left(11x^2+2y\right)\mathbf{j}+\left(x^2-4z\right)\mathbf{k},\ S:\ \begin{cases} x+y+2z=2, \\ x=0,\ y=0,\ z=0. \end{cases}$

7.26. $\mathbf{a}=\left(y^2+z^2+6x\right)\mathbf{i}+\left(\mathrm{e}^{z}-2y+x\right)\mathbf{j}+\left(x+y-z\right)\mathbf{k},\ S:\ \begin{cases} x^2+y^2=z^2, \\ z=1,\ z=3. \end{cases}$

7.27. $\mathbf{a}=\dfrac{1}{2}\left(x+z\right)\mathbf{i}+\dfrac{1}{4}\left(x\cdot z+y\right)\mathbf{j}+\left(xy-2\right)\mathbf{k},\ S:\ x^2+y^2+z^2=4x-2y+4z-8.$

7.28. $\mathbf{a}=\left(3yz-x\right)\mathbf{i}+\left(x^2-y\right)\mathbf{j}+\left(6z-1\right)\mathbf{k},\ S:\ \begin{cases} z^2=9\left(x^2+y^2\right), \\ z=3. \end{cases}$

7.29. $\mathbf{a}=(yz-2x)\mathbf{i}+(\sin x+y)\mathbf{j}+(x-2z)\mathbf{k},\ S:\ \begin{cases} x+2y-3z=6, \\ x=0,\ y=0,\ z=0. \end{cases}$

7.30. $\mathbf{a}=(8x+1)\mathbf{i}+(zx-4y)\mathbf{j}+(\mathrm{e}^{x}-z)\mathbf{k},\ S:\ x^2+y^2+z^2=2y.$

7.31. $\mathbf{a}=(2y-5x)\mathbf{i}+(x-1)\mathbf{j}+(2\sqrt{xy}+2z)\mathbf{k},\ S:\ \begin{cases} 2x+2y-z=4, \\ x=0,\ y=0,\ z=0. \end{cases}$

Задача 8. Найти поток векторного поля $\mathbf{a}$ через замкнутую поверхность S (нормаль внешняя).

8.1. $\mathbf{a}=(x+z)\mathbf{i}+(z+y)\mathbf{k},$ $S:\ \begin{cases} x^2+y^2=9, \\ z=x,\ z=0\ (z\geq 0). \end{cases}$

8.2. $\mathbf{a}=2x\mathbf{i}+z\mathbf{k},$ $S:\ \begin{cases} z=3x^2+2y^2+1, \\ x^2+y^2=4,\ z=0. \end{cases}$

8.3. $\mathbf{a}=2x\mathbf{i}+2y\mathbf{j}+z\mathbf{k},$ $S:\ \begin{cases} y=x^2,\ y=4x^2,\ y=1\ (x\geq 0) \\ z=y,\ z=0. \end{cases}$

8.4. $\mathbf{a}=3x\mathbf{i}-z\mathbf{j},$ $S:\ \begin{cases} z=6-x^2-y^2, \\ z^2=x^2+y^2\,(z\geq 0). \end{cases}$

8.5. $\mathbf{a}=(z+y)\mathbf{i}+y\mathbf{j}-x\mathbf{k},$ $S:\ \begin{cases} x^2+y^2=2y, \\ y=2. \end{cases}$

8.6. $\mathbf{a}=x\mathbf{i}-(x+2y)\mathbf{j}+y\mathbf{k},$ $S:\ \begin{cases} x^2+y^2=1,\ z=0, \\ x+2y+3z=6. \end{cases}$

8.7. $\mathbf{a}=2(z-y)\mathbf{j}+(x-z)\mathbf{k},$ $S:\ \begin{cases} z=x^2+3y^2+1,\ z=0, \\ x^2+y^2=1. \end{cases}$

8.8. $\mathbf{a}=x\mathbf{i}+z\mathbf{j}-y\mathbf{k},$ $S:\ \begin{cases} z=4-2(x^2+y^2), \\ z=2(x^2+y^2). \end{cases}$

8.9. $\mathbf{a}=z\mathbf{i}-4y\mathbf{j}+2x\mathbf{k},$ $S:\ \begin{cases} z=x^2+y^2, \\ z=1. \end{cases}$

8.10. $\mathbf{a}=4x\mathbf{i}-2y\mathbf{j}-z\mathbf{k},$ $S:\ \begin{cases} 3x+2y=12,\ 3x+y=6,\ y=0, \\ x+y+z=6,\ z=0. \end{cases}$

8.11. $\mathbf{a}=x\mathbf{i}-2y\mathbf{j}+x\mathbf{k},$ $S:\ \begin{cases} x+y=1,\ x=0,\ y=0, \\ z=x^2+y^2,\ z=0. \end{cases}$

8.12. $\mathbf{a}=z\mathbf{i}+x\mathbf{j}-z\mathbf{k},$ $S:\ \begin{cases} 4z=x^2+y^2, \\ z=4. \end{cases}$

8.13. $\mathbf{a} = 6x\mathbf{i} - 2y\mathbf{j} - z\mathbf{k}$,
$$S: \begin{cases} z = 3 - 2\left(x^2 + y^2\right), \\ z = x^2 + y^2 \ \ (z \geq 0). \end{cases}$$

8.14. $\mathbf{a} = (z + y)\mathbf{i} + (x - z)\mathbf{j} + z\mathbf{k}$,
$$S: \begin{cases} x^2 + 4y^2 = 4, \\ 3x + 4y + z = 12, \ z = 1. \end{cases}$$

8.15. $\mathbf{a} = (y + 2z)\mathbf{i} - y\mathbf{j} + 3x\mathbf{k}$,
$$S: \begin{cases} 3z = 27 - 2\left(x^2 + y^2\right), \\ z^2 = x^2 + y^2, \ (z \geq 0). \end{cases}$$

8.16. $\mathbf{a} = (y + 6x)\mathbf{i} + 5(x + z)\mathbf{j} + 4y\mathbf{k}$,
$$S: \begin{cases} y = x, \ y = 2x, \ y = 2, \\ z = x^2 + y^2, \ z = 0. \end{cases}$$

8.17. $\mathbf{a} = y\mathbf{i} + 5y\mathbf{j} + z\mathbf{k}$,
$$S: \begin{cases} x^2 + y^2 = 1, \\ z = x, \ z = 0 \ \ (z \geq 0). \end{cases}$$

8.18. $\mathbf{a} = z\mathbf{i} + (3y - x)\mathbf{j} - z\mathbf{k}$,
$$S: \begin{cases} x^2 + y^2 = 1, \\ z = x^2 + y^2 + 2, \ z = 0. \end{cases}$$

8.19. $\mathbf{a} = y\mathbf{i} + (x + 2y)\mathbf{j} + x\mathbf{k}$,
$$S: \begin{cases} x^2 + y^2 = 2x, \\ z = x^2 + y^2, \\ z = 0. \end{cases}$$

8.20. $\mathbf{a} = (x + y + z)\mathbf{i} + (2y - x)\mathbf{j} + (3z + y)\mathbf{k}$,
$$S: \begin{cases} y = x, \ y = 2x, \ x = 1, \\ z = x^2 + y^2, \\ z = 0. \end{cases}$$

8.21. $\mathbf{a} = 7x\mathbf{i} + z\mathbf{j} + (x - y + 5z)\mathbf{k}$,
$$S: \begin{cases} z = x^2 + y^2, \\ z = x^2 + 2y^2, \\ y = x, \ y = 2x, \ x = 1. \end{cases}$$

8.22. $\mathbf{a} = 17x\mathbf{i} + 7y\mathbf{j} + 11z\mathbf{k}$,
$$S: \begin{cases} z = x^2 + y^2, \\ z = 2\left(x^2 + y^2\right), \\ y = x^2, \ y = x. \end{cases}$$

8.23. $\mathbf{a} = x\mathbf{i} - 2y\mathbf{j} + 3z\mathbf{k}$,
$$S: \begin{cases} x^2 + y^2 = z, \\ z = 2x. \end{cases}$$

8.24. $\mathbf{a} = (2x + y)\mathbf{i} + (y + 2z)\mathbf{k}$,
$$S: \begin{cases} z = 2 - 4\left(x^2 + y^2\right), \\ z = 4\left(x^2 + y^2\right). \end{cases}$$

8.25. $\mathbf{a} = (2y - 3z)\mathbf{i} + (3x + 2z)\mathbf{j} + (x + y + z)\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = 1, \\ z = 4 - x - y, z = 0. \end{cases}$$

8.26. $\mathbf{a} = -2x\mathbf{i} + z\mathbf{j} + (x + y)\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = 2y, \\ z = x^2 + y^2, z = 0. \end{cases}$$

8.27. $\mathbf{a} = (2y - 15x)\mathbf{i} + (z - y)\mathbf{j} - (x - 3y)\mathbf{k},$

$$S: \begin{cases} z = 3x^2 + y^2 + 1, \ z = 0, \\ x^2 + y^2 = \dfrac{1}{4}. \end{cases}$$

8.28. $\mathbf{a} = (y + z)\mathbf{i} + (x - 2y + z)\mathbf{j} + x\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = 1, \\ z = x^2 + y^2, z = 0. \end{cases}$$

8.29. $\mathbf{a} = (3x - y - z)\mathbf{i} + 3y\mathbf{j} + 2z\mathbf{k},$

$$S: \{ z = x^2 + y^2, \ z = 2y.$$

8.30. $\mathbf{a} = (x + y)\mathbf{i} + (y + z)\mathbf{j} + (z + x)\mathbf{k},$

$$S: \begin{cases} y = 2x, \ y = 4x, \ x = 1, \\ z = y^2, z = 0. \end{cases}$$

8.31. $\mathbf{a} = (x + z)\mathbf{i} + y\mathbf{k},$

$$S: \begin{cases} z = 8 - x^2 - y^2, \\ z = x^2 + y^2. \end{cases}$$

Задача 9. Найти поток векторного поля $\mathbf{a}$ через замкнутую поверхность S (нормаль внешняя).

9.1. $\mathbf{a} = x^2\mathbf{i} + x\mathbf{j} + xz\mathbf{k},$

$$S: \begin{cases} z = x^2 + y^2, \ z = 1, \\ x = 0, \ y = 0 \ (1 \text{ октант}). \end{cases}$$

9.2. $\mathbf{a}=\left(x^2+y^2\right)\mathbf{i}+\left(x^2+y^2\right)\mathbf{j}+\left(x^2+y^2\right)\mathbf{k},$

$S:\ \begin{cases} z=x^2+y^2, \\ z=0,\ z=1. \end{cases}$

9.3. $\mathbf{a}=x^2\mathbf{i}+y^2\mathbf{j}+z^2\mathbf{k},$

$S:\ \begin{cases} x^2+y^2+z^2=4, \\ x^2+y^2=z^2\ \ (z\ge 0). \end{cases}$

9.4. $\mathbf{a}=x^2\mathbf{i}+y\mathbf{j}+z\mathbf{k},$

$S:\ \begin{cases} x^2+y^2+z^2=1, \\ z=0\ \ (z\ge 0). \end{cases}$

9.5. $\mathbf{a}=xz\mathbf{i}+z\mathbf{j}+y\mathbf{k},$

$S:\ \begin{cases} x^2+y^2=1-z, \\ z=0. \end{cases}$

9.6. $\mathbf{a}=3xz\mathbf{i}-2x\mathbf{j}+y\mathbf{k},$

$S:\ \begin{cases} x+y+z=2,\ x=1, \\ x=0,\ y=0,\ z=0. \end{cases}$

9.7. $\mathbf{a}=x^2\mathbf{i}+y^2\mathbf{j}+z^2\mathbf{k},$

$S:\ \begin{cases} x^2+y^2+z^2=2, \\ z=0\ \ (z\ge 0). \end{cases}$

9.8. $\mathbf{a}=x^3\mathbf{i}+y^3\mathbf{j}+z^3\mathbf{k},$

$S:\ x^2+y^2+z^2=1.$

9.9. $\mathbf{a}=\left(zx+y\right)\mathbf{i}+\left(zy-x\right)\mathbf{j}-\left(x^2+y^2\right)\mathbf{k},$

$S:\ \begin{cases} x^2+y^2+z^2=1, \\ z=0\ \ (z\ge 0). \end{cases}$

9.10. $\mathbf{a}=y^2x\mathbf{i}+z^2y\mathbf{j}+x^2z\mathbf{k},$

$S:\ x^2+y^2+z^2=1.$

9.11. $\mathbf{a}=x^2\mathbf{i}+y^2\mathbf{j}+z^2\mathbf{k},$

$S:\ \begin{cases} x^2+y^2+z^2=1, \\ x=0,\ y=0,\ z=0 \\ (1\ \text{октант}). \end{cases}$

9.12. $\mathbf{a}=x^2\mathbf{i}+xy\mathbf{j}+3z\mathbf{k},$

$S:\begin{cases} x^2+y^2=z^2, \\ z=4. \end{cases}$

9.13. $\mathbf{a}=\left(zx+y\right)\mathbf{i}+\left(xy-z\right)\mathbf{j}+\left(x^2+yz\right)\mathbf{k},$

$S:\ \begin{cases} x^2+y^2=2, \\ z=0,\ z=1. \end{cases}$

9.14. $\mathbf{a} = xy^2\mathbf{i} + x^2y\mathbf{j} + z\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = 1,\ z = 0,\ z = 1, \\ x = 0,\ y = 0 \\ (1\ \text{октант}). \end{cases}$$

9.15. $\mathbf{a} = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 + z^2 = 16, \\ x^2 + y^2 = z^2\ \ (z \ge 0). \end{cases}$$

9.16. $\mathbf{a} = 3x^2\mathbf{i} - 2x^2y\mathbf{j} + (2x - 1)z\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = 1, \\ z = 0,\ z = 1. \end{cases}$$

9.17. $\mathbf{a} = x^2\mathbf{i} + y^2\mathbf{j} + 2z\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = \dfrac{1}{4}, \\ z = 0,\ z = 2. \end{cases}$$

9.18. $\mathbf{a} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = 4, \\ z = 0,\ z = 1. \end{cases}$$

9.19. $\mathbf{a} = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ x = 0,\ y = 0,\ z = 0 \\ (1\ \text{октант}). \end{cases}$$

9.20. $\mathbf{a} = z\mathbf{i} + yz\mathbf{j} - xy\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = 4, \\ z = 0,\ z = 1. \end{cases}$$

9.21. $\mathbf{a} = (zx + y)\mathbf{i} - (2y - x)\mathbf{j} - (x^2 + y^2)\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ z = 0\ \ (z \ge 0). \end{cases}$$

9.22. $\mathbf{a} = (x^2 + xy)\mathbf{i} + (y^2 + yz)\mathbf{j} + (z^2 + xz)\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ x^2 + y^2 = z^2\ \ (z \ge 0). \end{cases}$$

9.23. $\mathbf{a} = 3x^2\mathbf{i} - 2x^2y\mathbf{j} - (1 - 2x)\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = 1, \\ z = 0,\ z = 1. \end{cases}$$

9.24. $\mathbf{a} = x^2\mathbf{i},$

$$S: \begin{cases} z = 1 - x - y, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$$

9.25. $\mathbf{a} = (y^2 + xz)\mathbf{i} + (yx - z)\mathbf{j} + (yz + x)\mathbf{k},$

$$S: \begin{cases} x^2 + y^2 = 1, \\ z = 0,\ z = \sqrt{2}. \end{cases}$$

$\mathbf{a} = y\mathbf{i} + y^2\mathbf{j} + yz\mathbf{k}$,

9.26. $S:\ \begin{cases} z = x^2 + y^2,\ z = 1, \\ x = 0,\ y = 0 \\ \ (1\ \text{октант}). \end{cases}$

$\mathbf{a} = y\mathbf{i} + 2zy\mathbf{j} + 2z^2\mathbf{k}$,

9.27. $S:\ \begin{cases} x^2 + y^2 = 1 - z, \\ z = 0. \end{cases}$

$\mathbf{a} = 2xy\mathbf{i} + 2xy\mathbf{j} + z^2\mathbf{k}$,

9.28. $S:\ \begin{cases} x^2 + y^2 + z^2 = \sqrt{2}, \\ z = 0\ \ (z \geq 0). \end{cases}$

$\mathbf{a} = y^2x\mathbf{i} + x^2y\mathbf{j} + z^3\mathbf{k}/3$,

9.29. $S:\ \begin{cases} x^2 + y^2 + z^2 = 1, \\ z = 0,\ (z \geq 0). \end{cases}$

$\mathbf{a} = -x\mathbf{i} + 2y\mathbf{j} + yz\mathbf{k}$,

9.30. $S:\ \begin{cases} x^2 + y^2 = z^2, \\ z = 4. \end{cases}$

$\mathbf{a} = \left(y^2 + z^2\right)\mathbf{i} + \left(xy + y^2\right)\mathbf{j} + \left(xz + z\right)\mathbf{k}$,

9.31. $S:\ \begin{cases} x^2 + y^2 = 1, \\ z = 0,\ z = 1. \end{cases}$

Задача 10. Найти работу силы $\mathbf{F}$ при перемещении вдоль линии L от точки M к точке N.

10.1. $\mathbf{F} = \left(x^2 - 2y\right)\mathbf{i} + \left(y^2 - 2x\right)\mathbf{j}$,
$L:$ отрезок MN,
$M(-4,0),\ N(0,2)$.

10.2. $\mathbf{F} = \left(x^2 + 2y\right)\mathbf{i} + \left(y^2 + 2x\right)\mathbf{j}$,
$L:$ отрезок MN,
$M(-4,0),\ N(0,2)$.

10.3. $\mathbf{F} = \left(x^2 + 2y\right)\mathbf{i} + \left(y^2 + 2x\right)\mathbf{j}$,
$L:\ 2 - \dfrac{x^2}{8} = y$,
$M(-4,0),\ N(0,2)$.

10.4. $\mathbf{F} = \left(x + y\right)\mathbf{i} + 2x\mathbf{j}$,
$L:\ x^2 + y^2 = 4\ \ (y \geq 0)$,
$M(2,0),\ N(-2,0)$.

10.5. $\mathbf{F} = x^3\mathbf{i} - y^3\mathbf{j}$,
$L:\ x^2 + y^2 = 4\ \ (x \geq 0,\ y \geq 0)$,
$M(2,0),\ N(0,2)$.

10.6. $\mathbf{F} = \left(x + y\right)\mathbf{i} + \left(x - y\right)\mathbf{j}$,
$L:\ y = x^2$,
$M(-1,1),\ N(1,1)$.

$\mathbf{F} = x^2 y\mathbf{i} - y\mathbf{j},$

10.7. $L:$ отрезок MN,

$M(-1,0),\ N(0,1).$

$\mathbf{F} = (2xy - y)\mathbf{i} + (x^2 + x)\mathbf{j},$

10.8. $L:\ x^2 + y^2 = 9\ (y \geq 0),$

$M(3,0),\ N(-3,0).$

$\mathbf{F} = (x + y)\mathbf{i} + (x - y)\mathbf{j},$

10.9. $L:\ x^2 + \dfrac{y^2}{9} = 1\ (x \geq 0,\ y \geq 0),$

$M(1,0),\ N(0,3).$

$\mathbf{F} = y\mathbf{i} - x\mathbf{j},$

10.10. $L:\ x^2 + y^2 = 1\ (y \geq 0),$

$M(1,0),\ N(-1,0).$

$\mathbf{F} = (x^2 + y^2)\mathbf{i} + (x^2 - y^2)\mathbf{j},$

10.11. $L:\ \begin{cases} x,\ 0 \leq x \leq 1; \\ 2 - x,\ 1 \leq x \leq 2; \end{cases}$

$M(2,0),\ N(0,0).$

$\mathbf{F} = y\mathbf{i} - x\mathbf{j},$

10.12. $L:\ x^2 + y^2 = 2\ (y \geq 0),$

$M(\sqrt{2},0),\ N(-\sqrt{2},0).$

$\mathbf{F} = xy\mathbf{i} + 2y\mathbf{j},$

10.13. $L:\ x^2 + y^2 = 1\ (x \geq 0,\ y \geq 0),$

$M(1,0),\ N(0,1).$

$\mathbf{F} = y\mathbf{i} - x\mathbf{j},$

10.14. $L:\ 2x^2 + y^2 = 1\ (y \geq 0),$

$M\left(\dfrac{1}{\sqrt{2}},0\right),\ N\left(-\dfrac{1}{\sqrt{2}},0\right).$

$\mathbf{F} = (x^2 + y^2)(\mathbf{i} + 2\mathbf{j}),$

10.15. $L:\ x^2 + y^2 = R^2\ (y \geq 0),$

$M(R,0),\ N(-R,0).$

$\mathbf{F} = \left(x + y\sqrt{x^2 + y^2}\right)\mathbf{i} + \left(y - x\sqrt{x^2 + y^2}\right)\mathbf{j},$

10.16. $L:\ x^2 + y^2 = 1\ (y \geq 0),$

$M(1,0),\ N(-1,0).$

$\mathbf{F} = x^2 y\mathbf{i} - xy^2\mathbf{j},$

10.17. $L:\ x^2 + y^2 = 4\ (x \geq 0,\ y \geq 0),$

$M(2,0),\ N(0,2).$

$\mathbf{F}=\left(x+y\sqrt{x^2+y^2}\right)\mathbf{i}+\left(y-\sqrt{x^2+y^2}\right)\mathbf{j},$

10.18. $L:\ x^2+y^2=16\ \left(x\ge 0,\ y\ge 0\right),$

$M(4,0),\ N(0,4).$

$\mathbf{F}=y^2\mathbf{i}-x^2\mathbf{j},$

10.19. $L:\ x^2+y^2=9\ \left(x\ge 0,\ y\ge 0\right),$

$M(3,0),\ N(0,3).$

$\mathbf{F}=\left(x+y\right)^2\mathbf{i}-\left(x^2+y^2\right)\mathbf{j},$

10.20. $L:$ отрезок MN,

$M(1,0),\ N(0,1).$

$\mathbf{F}=\left(x^2+y^2\right)\mathbf{i}+y^2\mathbf{j},$

10.21. $L:$ отрезок MN,

$M(2,0),\ N(0,2).$

$\mathbf{F}=x^2\mathbf{j},$

10.22. $L:\ x^2+y^2=9\ \left(x\ge 0,\ y\ge 0\right),$

$M(3,0),\ N(0,3).$

$\mathbf{F}=\left(y^2-y\right)\mathbf{i}+\left(2xy+x\right)\mathbf{j},$

10.23. $L:\ x^2+y^2=9\ \left(y\ge 0\right),$

$M(3,0),\ N(-3,0).$

$\mathbf{F}=xy\mathbf{i},$

10.24. $L:\ y=\sin x,$

$M(\pi,0),\ N(0,0).$

$\mathbf{F}=\left(xy-y^2\right)\mathbf{i}+x\mathbf{j},$

10.25. $L:\ \mathrm{y}=2x^2,$

$M(0,0),\ N(1,2).$

$\mathbf{F}=x\mathbf{i}+y\mathbf{j},$

10.26. $L:$ отрезок MN,

$M(1,0),\ N(0,3).$

$\mathbf{F}=\left(xy-x\right)\mathbf{i}+\dfrac{x^2}{2}\mathbf{j},$

10.27. $L:\ y=2\sqrt{x},$

$M(0,0),\ N(1,2).$

$\mathbf{F}=-x\mathbf{i}+y\mathbf{j},$

10.28. $L:\ x^2+\dfrac{y^2}{9}=1\ \left(x\ge 0,\ y\ge 0\right),$

$M(1,0),\ N(0,3).$

$\mathbf{F}=-y\mathbf{i}+x\mathbf{j},$

10.29. $L:\ y=x^3,$

$M(0,0),\ N(2,8).$

$\mathbf{F}=\left(x^2-y^2\right)\mathbf{i}+\left(x^2+y^2\right)\mathbf{j},$

10.30. $L:\ \dfrac{x^2}{9}+\dfrac{y^2}{4}=1\ \left(y\ge 0\right),$

$M(3,0),\ N(-3,0).$

$\mathbf{F}=(x-y)\mathbf{i}+\mathbf{j},$

10.31. $L:\ x^2+y^2=4\ (y\geq 0),$

$M(2,0),\ N(-2,0).$

Задача 11. Найти циркуляцию векторного поля $\mathbf{a}$ вдоль контура $\mathbf{\Gamma}$ (в направлении, соответствующем возрастанию параметра t).

11.1. $\mathbf{a}=y\mathbf{i}-x\mathbf{j}+z^2\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=\dfrac{\sqrt{2}}{2}\cos t,\ y=\dfrac{\sqrt{2}}{2}\cos t,\\ z=\sin t.\end{cases}$

11.2. $\mathbf{a}=-x^2y^3\mathbf{i}+\mathbf{j}+z\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=\sqrt[3]{4}\cos t,\ y=\sqrt[3]{4}\sin t,\\ z=3.\end{cases}$

11.3. $\mathbf{a}=(y-z)\mathbf{i}+(z-x)\mathbf{j}+(x-y)\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=\cos t,\ y=\sin t,\\ z=2(1-\cos t).\end{cases}$

11.4. $\mathbf{a}=x^2\mathbf{i}+y\mathbf{j}-z\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=\cos t,\ y=\left(\sqrt{2}\sin t\right)/2,\\ z=\left(\sqrt{2}\cos t\right)/2.\end{cases}$

11.5. $\mathbf{a}=(y-z)\mathbf{i}+(z-x)\mathbf{j}+(x-y)\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=4\cos t,\ y=4\sin t,\\ z=1-\cos t.\end{cases}$

11.6. $\mathbf{a}=2y\mathbf{i}-3x\mathbf{j}+x\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=2\cos t,\ y=2\sin t,\\ z=2-2\cos t-2\sin t.\end{cases}$

11.7. $\mathbf{a}=2z\mathbf{i}-x\mathbf{j}+y\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=2\cos t,\ y=2\sin t,\\ z=1.\end{cases}$

11.8. $\mathbf{a}=y\mathbf{i}-x\mathbf{j}+z\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=\cos t,\ y=\sin t,\\ z=3.\end{cases}$

11.9. $\mathbf{a}=x\mathbf{i}+z^2\mathbf{j}+y\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=\cos t,\ y=2\sin t,\\ z=2\cos t-2\sin t-1.\end{cases}$

11.10. $\mathbf{a}=3y\mathbf{i}-3x\mathbf{j}+x\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=3\cos t,\ y=3\sin t,\\ z=3-3\cos t-3\sin t.\end{cases}$

11.11. $\mathbf{a}=-x^2y^3\mathbf{i}+2\mathbf{j}+xz\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=\sqrt{2}\cos t,\ y=\sqrt{2}\sin t,\\ z=1.\end{cases}$

11.12. $\mathbf{a}=6z\mathbf{i}-x\mathbf{j}+xy\mathbf{k},$ $\mathbf{\Gamma}:\begin{cases}x=3\cos t,\ y=3\sin t,\\ z=3.\end{cases}$

11.13. $\mathbf{a} = z\mathbf{i} + y^2\mathbf{j} - x\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = \sqrt{2}\cos t,\ y = 2\sin t, \\ z = \sqrt{2}\cos t. \end{cases}$

11.14. $\mathbf{a} = x\mathbf{i} + 2z^2\mathbf{j} + y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = \cos t,\ y = 3\sin t, \\ z = 2\cos t - 3\sin t - 2. \end{cases}$

11.15. $\mathbf{a} = x\mathbf{i} - \frac{1}{3}z^2\mathbf{j} + y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = (\cos t)/2,\ y = (\sin t)/3, \\ z = \cos t - (\sin t)/3 - 1/4. \end{cases}$

11.16. $\mathbf{a} = 4y\mathbf{i} - 3x\mathbf{j} + x\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 4\cos t,\ y = 4\sin t, \\ z = 4 - 4\cos t - 4\sin t. \end{cases}$

11.17. $\mathbf{a} = -z\mathbf{i} - x\mathbf{j} + xz\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 5\cos t,\ y = 5\sin t, \\ z = 4. \end{cases}$

11.18. $\mathbf{a} = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 2\cos t,\ y = 2\sin t, \\ z = 0. \end{cases}$

11.19. $\mathbf{a} = (y - z)\mathbf{i} + (z - x)\mathbf{j} + (x - y)\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 3\cos t,\ y = 3\sin t, \\ z = 2(1 - \cos t). \end{cases}$

11.20. $\mathbf{a} = 2y\mathbf{i} - z\mathbf{j} + x\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = \cos t,\ y = \sin t, \\ z = 4 - \cos t - \sin t. \end{cases}$

11.21. $\mathbf{a} = xz\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = \cos t,\ y = \sin t, \\ z = \sin t. \end{cases}$

11.22. $\mathbf{a} = -x^2y^3\mathbf{i} + 3\mathbf{j} + y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = \cos t,\ y = \sin t, \\ z = 5. \end{cases}$

11.23. $\mathbf{a} = 7z\mathbf{i} - x\mathbf{j} + yz\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 6\cos t,\ y = 6\sin t, \\ z = 1/3. \end{cases}$

11.24. $\mathbf{a} = xy\mathbf{i} + x\mathbf{j} + y^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = \cos t,\ y = \sin t, \\ z = \sin t. \end{cases}$

11.25. $\mathbf{a} = x\mathbf{i} - z^2\mathbf{j} + y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 2\cos t,\ y = 3\sin t, \\ z = 4\cos t - 3\sin t - 3. \end{cases}$

11.26. $\mathbf{a} = (y - z)\mathbf{i} + (z - x)\mathbf{j} + (x - y)\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 2\cos t,\ y = 2\sin t, \\ z = 3(1 - \cos t). \end{cases}$

11.27. $\mathbf{a} = -2z\mathbf{i} - x\mathbf{j} + x^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = (\cos t)/3,\ y = (\sin t)/3, \\ z = 8. \end{cases}$

11.28. $\mathbf{a} = x\mathbf{i} - 3z^2\mathbf{j} + y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = \cos t,\ y = 4\sin t, \\ z = 2\cos t - 4\sin t + 3. \end{cases}$

11.29. $\mathbf{a} = x\mathbf{i} - 2z^2\mathbf{j} + y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 3\cos t,\ y = 4\sin t, \\ z = 6\cos t - 4\sin t + 1. \end{cases}$

11.30. $\mathbf{a} = -x^2y^3\mathbf{i} + 4\mathbf{j} + x\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 2\cos t,\ y = 2\sin t, \\ z = 4. \end{cases}$

11.31. $\mathbf{a} = y\mathbf{i}/3 - 3x\mathbf{j} + x\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x = 2\cos t,\ y = 2\sin t, \\ z = 1 - 2\cos t - 2\sin t. \end{cases}$

Задача 12. Найти модуль циркуляции векторного поля $\mathbf{a}$ вдоль контура $\mathbf{\Gamma}$.

12.1. $\mathbf{a} = \left(x^2 - y\right)\mathbf{i} + x\mathbf{j} + \mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 1, \\ z = 1. \end{cases}$

12.2. $\mathbf{a} = xz\mathbf{i} - \mathbf{j} + y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} z = 5\left(x^2 + y^2\right) - 1, \\ z = 4. \end{cases}$

12.3. $\mathbf{a} = yz\mathbf{i} + 2xz\mathbf{j} + xy\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 + z^2 = 25, \\ x^2 + y^2 = 9\ \left(z > 0\right). \end{cases}$

12.4. $\mathbf{a} = x\mathbf{i} + yz\mathbf{j} - x\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 1, \\ x + y + z = 1. \end{cases}$

12.5. $\mathbf{a} = \left(x - y\right)\mathbf{i} + x\mathbf{j} - z\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 1, \\ z = 1. \end{cases}$

12.6. $\mathbf{a} = y\mathbf{i} - x\mathbf{j} + z^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} z = 3\left(x^2 + y^2\right) + 1, \\ z = 4. \end{cases}$

12.7. $\mathbf{a} = yz\mathbf{i} + 2xz\mathbf{j} + y^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 + z^2 = 25, \\ x^2 + y^2 = 16\ \left(z > 0\right). \end{cases}$

12.8. $\mathbf{a} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 9, \\ x + y + z = 1. \end{cases}$

12.9. $\mathbf{a} = y\mathbf{i} + \left(1 - x\right)\mathbf{j} - z\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 + z^2 = 4, \\ x^2 + y^2 = 1\ \left(z > 0\right). \end{cases}$

12.10. $\mathbf{a} = y\mathbf{i} - x\mathbf{j} + z^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 1, \\ z = 4. \end{cases}$

12.11. $\mathbf{a} = 4x\mathbf{i} + 2\mathbf{j} - xy\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} z = 2\left(x^2 + y^2\right) + 1, \\ z = 7. \end{cases}$

12.12. $\mathbf{a} = 2y\mathbf{i} - 3x\mathbf{j} + z^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = z, \\ z = 1. \end{cases}$

12.13. $\mathbf{a} = -3z\mathbf{i} + y^2\mathbf{j} + 2y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 4, \\ x - 3y - 2z = 1. \end{cases}$

12.14. $\mathbf{a} = 2y\mathbf{i} + 5z\mathbf{j} + 3x\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} 2x^2 + 2y^2 = 1, \\ x + y + z = 3. \end{cases}$

12.15. $\mathbf{a} = 2y\mathbf{i} + \mathbf{j} - 2yz\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 - z^2 = 0, \\ z = 2. \end{cases}$

12.16. $\mathbf{a} = (x - y)\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 - 4z^2 = 0, \\ z = \dfrac{1}{2}. \end{cases}$

12.17. $\mathbf{a} = xz\mathbf{i} - \mathbf{j} + y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 + z^2 = 4, \\ z = 1. \end{cases}$

12.18. $\mathbf{a} = 2yz\mathbf{i} + xz\mathbf{j} - x^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 + z^2 = 25, \\ x^2 + y^2 = 9 \ (z > 0). \end{cases}$

12.19. $\mathbf{a} = 4x\mathbf{i} - yz\mathbf{j} + x\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 1, \\ x + y + z = 1. \end{cases}$

12.20. $\mathbf{a} = -y\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 - z^2 = 0, \\ z = 1. \end{cases}$

12.21. $\mathbf{a} = y\mathbf{i} + 3x\mathbf{j} + z^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} z = x^2 + y^2 - 1, \\ z = 3. \end{cases}$

12.22. $\mathbf{a} = 2yz\mathbf{i} + xz\mathbf{j} + y^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 + z^2 = 25, \\ x^2 + y^2 = 16 \ (z > 0). \end{cases}$

12.23. $\mathbf{a} = (2 - xy)\mathbf{i} - yz\mathbf{j} - xz\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 4, \\ x + y + z = 1. \end{cases}$

12.24. $\mathbf{a} = -y\mathbf{i} + x\mathbf{j} + 3z^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 + z^2 = 9, \\ x^2 + y^2 = 1 \ (z > 0). \end{cases}$

12.25. $\mathbf{a} = y\mathbf{i} - x\mathbf{j} + 2z\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 - \dfrac{z^2}{4} = 0, \\ z = 2. \end{cases}$

12.26. $\mathbf{a} = x^2\mathbf{i} + yz\mathbf{j} + 2z\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 + z^2 = 25, \\ z = 4. \end{cases}$

12.27. $\mathbf{a} = y\mathbf{i} - 2x\mathbf{j} + z^2\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} z = 4\left(x^2 + y^2\right) + 2, \\ z = 6. \end{cases}$

12.28. $\mathbf{a} = 3z\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 4, \\ 2x - 3y - 2z = 1. \end{cases}$

12.29. $\mathbf{a} = \left(x + y\right)\mathbf{i} - x\mathbf{j} + 6\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 = 1, \\ z = 2. \end{cases}$

12.30. $\mathbf{a} = 4\mathbf{i} + 3x\mathbf{j} + 3xz\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 - z^2 = 0, \\ z = 3. \end{cases}$

12.31. $\mathbf{a} = yz\mathbf{i} - xz\mathbf{j} + xy\mathbf{k}$, $\mathbf{\Gamma}: \begin{cases} x^2 + y^2 + z^2 = 9, \\ x^2 + y^2 = 9. \end{cases}$