VII. КРАТНЫЕ ИНТЕГРАЛЫ

Расчетные задания

Задача 1. Изменить порядок интегрирования.

1.1. $\displaystyle\int_{-2}^{-1} dy \int_{-\sqrt{2+y}}^{0} f\,dx + \int_{-1}^{0} dy \int_{-\sqrt{-y}}^{0} f\,dx$.

1.2. $\displaystyle\int_{0}^{1} dy \int_{-\sqrt{y}}^{0} f\,dx + \int_{1}^{\sqrt{2}} dy \int_{-\sqrt{-y}}^{0} f\,dx$.

1.3. $\displaystyle\int_{0}^{1} dy \int_{0}^{y} f\,dx + \int_{1}^{\sqrt{2}} dy \int_{0}^{\sqrt{2-y^2}} f\,dx$.

1.4. $\displaystyle\int_{0}^{1} dy \int_{0}^{\sqrt{y}} f\,dx + \int_{1}^{2} dy \int_{0}^{\sqrt{2-y}} f\,dx$.

1.5. $\displaystyle\int_{-\sqrt{2}}^{-1} dx \int_{-\sqrt{2-x^2}}^{0} f\,dy + \int_{-1}^{0} dx \int_{x}^{0} f\,dy$.

1.6. $\displaystyle\int_{0}^{1/\sqrt{2}} dy \int_{0}^{\arcsin y} f\,dx + \int_{1/\sqrt{2}}^{1} dy \int_{0}^{\arccos y} f\,dx$.

1.7. $\displaystyle\int_{-2}^{-1} dy \int_{0}^{\sqrt{2+y}} f\,dx + \int_{-1}^{0} dy \int_{0}^{\sqrt{-y}} f\,dx$.

1.8. $\displaystyle\int_{0}^{1} dy \int_{-\sqrt{y}}^{0} f\,dx + \int_{1}^{e} dy \int_{-1}^{-\ln y} f\,dx$.

1.9. $\displaystyle\int_{-\sqrt{2}}^{-1} dx \int_{0}^{\sqrt{2-x^2}} f\,dy + \int_{-1}^{0} dx \int_{0}^{x^2} f\,dy$.

1.10. $\displaystyle\int_{-2}^{-\sqrt{3}} dx \int_{-\sqrt{4-x^2}}^{0} f\,dy + \int_{-\sqrt{3}}^{0} dx \int_{\sqrt{4-x^2}-2}^{0} f\,dy$.

1.11. $\displaystyle\int_{0}^{1} dx \int_{1-x^2}^{1} f\,dy + \int_{1}^{e} dx \int_{\ln x}^{1} f\,dy$.

1.12. $\displaystyle\int_{0}^{1} dy \int_{0}^{\sqrt[3]{y}} f\,dx + \int_{1}^{2} dy \int_{0}^{2-y} f\,dx$.

1.13. $\displaystyle\int_{0}^{\pi/4} dy \int_{0}^{\sin y} f\,dx + \int_{\pi/4}^{\pi/2} dy \int_{0}^{\cos y} f\,dx.$.

1.14. $\displaystyle\int_{-2}^{-1} dx \int_{-(2+x)}^{0} f\,dy + \int_{-1}^{0} dx \int_{\sqrt[3]{x}}^{0} f\,dy$.

1.15. $\displaystyle\int_{0}^{1} dy \int_{0}^{\sqrt{y}} f\,dx + \int_{1}^{e} dy \int_{\ln y}^{1} f\,dx$.

1.16. $\displaystyle\int_{0}^{1} dy \int_{-\sqrt{y}}^{0} f\,dx + \int_{1}^{2} dy \int_{-\sqrt{2-y}}^{0} f\,dx$.

1.17. $\displaystyle\int_{0}^{1} dy \int_{-y}^{0} f\,dx + \int_{1}^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^{0} f\,dx$.

1.18. $\displaystyle\int_{0}^{1} dy \int_{0}^{y^2} f\,dx + \int_{1}^{2} dy \int_{0}^{2-y} f\,dx$.

1

1.19. $\displaystyle\int_0^{\sqrt{3}} dx \int_{\sqrt{4-x^2}-2}^{0} f\,dy + \int_{\sqrt{3}}^{2} dx \int_{-\sqrt{4-x^2}}^{0} f\,dy$.

1.20. $\displaystyle\int_{-2}^{-1} dy \int_{-(2+y)}^{0} f\,dx + \int_{-1}^{0} dy \int_{\sqrt[3]{y}}^{0} f\,dx$.

1.21. $\displaystyle\int_0^{1} dy \int_0^{y} f\,dx + \int_1^{e} dy \int_{\ln y}^{1} f\,dx$.

1.22. $\displaystyle\int_0^{1} dx \int_0^{x^2} f\,dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f\,dy$.

1.23. $\displaystyle\int_0^{\pi/4} dx \int_0^{\sin x} f\,dy + \int_{\pi/4}^{\pi/2} dx \int_0^{\cos x} f\,dy$.

1.24. $\displaystyle\int_{-\sqrt{2}}^{-1} dy \int_{-\sqrt{2-y^2}}^{0} f\,dx + \int_{-1}^{0} dy \int_{y}^{0} f\,dx$.

1.25. $\displaystyle\int_0^{1} dx \int_0^{x^2} f\,dy + \int_1^{2} dx \int_0^{2-x} f\,dy$.

1.26. $\displaystyle\int_0^{\sqrt{3}} dx \int_0^{2-\sqrt{4-x^2}} f\,dy + \int_{\sqrt{3}}^{2} dx \int_0^{\sqrt{4-x^2}} f\,dy$.

1.27. $\displaystyle\int_0^{1} dx \int_{-\sqrt{x}}^{0} f\,dy + \int_1^{2} dx \int_{-\sqrt{2-x}}^{0} f\,dy$.

1.28. $\displaystyle\int_0^{1} dx \int_0^{x} f\,dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f\,dy$.

1.29. $\displaystyle\int_0^{1} dy \int_0^{\sqrt{y}} f\,dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f\,dx$.

1.30. $\displaystyle\int_0^{1} dx \int_0^{\sqrt{x}} f\,dy + \int_1^{2} dx \int_0^{\sqrt{2-x}} f\,dy$.

1.31. $\displaystyle\int_{-2}^{-\sqrt{3}} dx \int_0^{\sqrt{4-x^2}} f\,dy + \int_{-\sqrt{3}}^{0} dx \int_0^{2-\sqrt{4-x^2}} f\,dy$.

Задача 2. Вычислить.

2.1. $\displaystyle\iint_D \left(12x^2 y^2 + 16x^3 y^3\right) dxdy;$

$D: x=1, y=x^2, y=-\sqrt{x}.$

2.2. $\displaystyle\iint_D \left(9x^2 y^2 + 48x^3 y^3\right) dxdy;$

$D: x=1, y=\sqrt{x}, y=-x^2.$

2.3. $\displaystyle\iint_D \left(36x^2 y^2 - 96x^3 y^3\right) dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^3.$

2.4. $\displaystyle\iint_D \left(18x^2 y^2 + 32x^3 y^3\right) dxdy;$

$D: x=1, y=x^3, y=-\sqrt[3]{x}.$

2.5. $\displaystyle\iint_D \left(27x^2 y^2 + 48x^3 y^3\right) dxdy;$

$D: x=1, y=x^2, y=-\sqrt[3]{x}.$

2.6. $\displaystyle\iint_D \left(18x^2 y^2 + 32x^3 y^3\right) dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^2.$

2.7. $\iint\limits_{D}\left(18x^2y^2 + 32x^3y^3\right)dxdy;$

$D: x=1, y=x^3, y=-\sqrt{x}.$

2.8. $\iint\limits_{D}\left(27x^2y^2 + 48x^3y^3\right)dxdy;$

$D: x=1, y=\sqrt{x}, y=-x^3.$

2.9. $\iint\limits_{D}\left(4xy + 3x^2y^2\right)dxdy;$

$D: x=1, y=x^2, y=-\sqrt{x}.$

2.10. $\iint\limits_{D}\left(12xy + 9x^2y^2\right)dxdy;$

$D: x=1, y=\sqrt{x}, y=-x^2.$

2.11. $\iint\limits_{D}\left(8xy + 9x^2y^2\right)dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^3.$

2.12. $\iint\limits_{D}\left(24xy + 18x^2y^2\right)dxdy;$

$D: x=1, y=x^3, y=-\sqrt[3]{x}.$

2.13. $\iint\limits_{D}\left(12xy + 27x^2y^2\right)dxdy;$

$D: x=1, y=x^2, y=-\sqrt[3]{x}.$

2.14. $\iint\limits_{D}\left(8xy + 18x^2y^2\right)dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^2.$

2.15. $\iint\limits_{D}\left(\dfrac{4}{5}xy + \dfrac{9}{11}x^2y^2\right)dxdy;$

$D: x=1, y=x^3, y=-\sqrt{x}.$

2.16. $\iint\limits_{D}\left(\dfrac{4}{5}xy + 9x^2y^2\right)dxdy;$

$D: x=1, y=\sqrt{x}, y=-x^3.$

2.17. $\iint\limits_{D}\left(24xy - 48x^3y^3\right)dxdy;$

$D: x=1, y=x^2, y=-\sqrt{x}.$

2.18. $\iint\limits_{D}\left(6xy + 24x^3y^3\right)dxdy;$

$D: x=1, y=\sqrt{x}, y=-x^2.$

2.19. $\iint\limits_{D}\left(4xy + 16x^3y^3\right)dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^3.$

2.20. $\iint\limits_{D}\left(4xy + 16x^3y^3\right)dxdy;$

$D: x=1, y=x^3, y=-\sqrt[3]{x}.$

2.21. $\iint\limits_{D}\left(44xy + 16x^3y^3\right)dxdy;$

$D: x=1, y=x^2, y=-\sqrt[3]{x}.$

2.22. $\iint\limits_{D}\left(4xy + 176x^3y^3\right)dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^3.$

2.23. $\iint\limits_{D}\left(xy - 4x^3y^3\right)dxdy;$

$D: x=1, y=x^3, y=-\sqrt{x}.$

2.24. $\iint\limits_{D}\left(4xy + 176x^3y^3\right)dxdy;$

$D: x=1, y=\sqrt{x}, y=-x^3.$

2.25. $\iint\limits_D \left(6x^2y^2 + \dfrac{25}{3}x^4y^4\right)dxdy;$

$D: x=1, \ y=x^2, \ y=-\sqrt{x}.$

2.26. $\iint\limits_D \left(9x^2y^2 + 25x^4y^4\right)dxdy;$

$D: x=1, \ y=\sqrt{x}, \ y=-x^2.$

2.27. $\iint\limits_D \left(3x^2y^2 + \dfrac{50}{3}x^4y^4\right)dxdy;$

$D: x=1, \ y=\sqrt[3]{x}, \ y=-x^3.$

2.28. $\iint\limits_D \left(9x^2y^2 + 25x^4y^4\right)dxdy;$

$D: x=1, \ y=x^3, \ y=-\sqrt[3]{x}.$

2.29. $\iint\limits_D \left(54x^2y^2 + 150x^4y^4\right)dxdy;$

$D: x=1, \ y=x^2, \ y=-\sqrt[3]{x}.$

2.30. $\iint\limits_D \left(xy - 9x^5y^5\right)dxdy;$

$D: x=1, \ y=\sqrt[3]{x}, \ y=-x^2.$

2.31. $\iint\limits_D \left(54x^2y^2 + 150x^4y^4\right)dxdy;$

$D: x=1, \ y=x^3, \ y=-\sqrt{x}.$

Задача 3. Вычислить.

3.1. $\iint\limits_D ye^{xy/2}dxdy;$

$D: y=\ln 2, \ y=\ln 3, \ x=2, \ x=4.$

3.2. $\iint\limits_D y^2 \sin\dfrac{xy}{2} \ dxdy;$

$D: x=0, \ y=\sqrt{\pi}, \ y=\dfrac{x}{2}.$

3.3. $\iint\limits_D y\cos xy \ dxdy;$

$D: y=\pi/2, \ y=\pi, \ x=1, \ x=2.$

3.4. $\iint\limits_D y^2 e^{-xy/4}dxdy;$

$D: x=0, \ y=2, \ y=x.$

3.5. $\iint\limits_D y\sin xy \ dxdy;$

$D: y=\pi/2, \ y=\pi, \ x=1, \ x=2.$

3.6. $\iint\limits_D y^2 \cos\dfrac{xy}{2} \ dxdy;$

$D: x=0, \ y=\sqrt{\pi/2}, \ y=x/2.$

3.7. $\iint\limits_D 4ye^{2xy}dxdy;$

$D: y=\ln 3, \ y=\ln 4, \ x=\dfrac{1}{2}, \ x=1.$

3.8. $\iint\limits_D 4y^2 \sin xy \ dxdy;$

$D: x=0, \ y=\sqrt{\dfrac{\pi}{2}}, \ y=x.$

3.9. $\iint\limits_{D} y\cos 2xy\, dxdy;$

$D:\ y=\dfrac{\pi}{2},\ y=\pi,\ x=\dfrac{1}{2},\ x=1.$

3.10. $\iint\limits_{D} y^2 e^{-xy/8}\, dxdy;$

$D:\ x=0,\ y=2,\ y=\dfrac{x}{2}.$

3.11. $\iint\limits_{D} 12y\sin 2xy\, dxdy;$

$D:\ y=\dfrac{\pi}{4},\ y=\dfrac{\pi}{2},\ x=2,\ x=3.$

3.12. $\iint\limits_{D} y^2\cos xy\, dxdy;$

$D:\ x=0,\ y=\sqrt{\pi},\ y=x.$

3.13. $\iint\limits_{D} ye^{xy/4}\, dxdy;$

$D:\ y=\ln 2,\ y=\ln 3,\ x=4,\ x=8.$

3.14. $\iint\limits_{D} y^2\sin 2xy\, dxdy;$

$D:\ x=0,\ y=\sqrt{2\pi},\ y=2x.$

3.15. $\iint\limits_{D} 2y\cos 2xy\, dxdy;$

$D:\ y=\dfrac{\pi}{4},\ y=\dfrac{\pi}{2},\ x=1,\ x=2.$

3.16. $\iint\limits_{D} y^2 e^{-xy/2}\, dxdy;$

$D:\ x=0,\ y=\sqrt{2},\ y=x.$

3.17. $\iint\limits_{D} y\sin xy\, dxdy;$

$D:\ y=\pi,\ y=2\pi,\ x=\dfrac{1}{2},\ x=1.$

3.18. $\iint\limits_{D} y^2\cos 2xy\, dxdy;$

$D:\ x=0,\ y=\sqrt{\dfrac{\pi}{2}},\ y=\dfrac{x}{2}.$

3.19. $\iint\limits_{D} 8ye^{4xy}\, dxdy;$

$D:\ y=\ln 3,\ y=\ln 4,\ x=\dfrac{1}{4},\ x=\dfrac{1}{2}.$

3.20. $\iint\limits_{D} 3y^2\sin\dfrac{xy}{2}\, dxdy;$

$D:\ x=0,\ y=\sqrt{\dfrac{4\pi}{3}},\ y=\dfrac{2}{3}x.$

3.21. $\iint\limits_{D} y\cos xy\, dxdy;$

$D:\ y=\pi,\ y=3\pi,\ x=1/2,\ x=1.$

3.22. $\iint\limits_{D} y^2 e^{-xy/2}\, dxdy;$

$D:\ x=0,\ y=1,\ y=\dfrac{x}{2}.$

3.23. $\iint\limits_{D} y\sin 2xy\, dxdy;$

$D:\ y=\pi/2,\ y=3\pi/2,\ x=1/2,\ x=2.$

3.24. $\iint\limits_{D} y^2\cos xy\, dxdy;$

$D:\ x=0,\ y=\sqrt{\pi},\ y=2x.$

3.25. $\iint\limits_{D} 6ye^{xy/3}dxdy;$

$D: y=\ln 2,\ y=\ln 3,\ x=3,\ x=6.$

3.26. $\iint\limits_{D} y^2 \sin\dfrac{xy}{2}\ dxdy;$

$D: x=0,\ y=\sqrt{\pi},\ y=x.$

3.27. $\iint\limits_{D} y\cos 2xy\ dxdy;$

$D: y=\pi/2,\ y=3\pi/2,\ x=1/2,\ x=2.$

3.28. $\iint\limits_{D} y^2 e^{-xy/8}dxdy;$

$D: x=0,\ y=4,\ y=2x.$

3.29. $\iint\limits_{D} 3y\sin xy\ dxdy;$

$D: y=\pi/2,\ y=3\pi,\ x=1,\ x=3.$

3.30. $\iint\limits_{D} y^2 \cos\dfrac{xy}{2}\ dxdy;$

$D: x=0,\ y=\sqrt{2\pi},\ y=2x.$

3.31. $\iint\limits_{D} 12ye^{6xy}dxdy;$

$D: y=\ln 3,\ y=\ln 4,\ x=1/6,\ x=1/3.$

Задача 4. Вычислить.

4.1. $\iiint\limits_{V} 2y^2 e^{xy}\ dx\,dy\,dz;$

$V \begin{cases} x=0,\ y=1,\ y=x, \\ z=0,\ z=1. \end{cases}$

4.2. $\iiint\limits_{V} x^2 z\sin(xyz)\ dx\,dy\,dz;$

$V \begin{cases} x=2,\ y=\pi,\ z=1, \\ x=0,\ y=1,\ z=0. \end{cases}$

4.3. $\iiint\limits_{V} y^2 \mathrm{ch}(2xy)\ dx\,dy\,dz;$

$V \begin{cases} x=0,\ y=-2,\ y=4x, \\ z=0,\ z=2. \end{cases}$

4.4. $\iiint\limits_{V} 8y^2 z\,e^{2xyz}\ dx\,dy\,dz;$

$V \begin{cases} x=-1,\ y=2,\ z=1, \\ x=0,\ y=0,\ z=0. \end{cases}$

4.5. $\iiint\limits_{V} x^2 \mathrm{sh}(3xy)\ dx\,dy\,dz;$

$V \begin{cases} x=1,\ y=2x,\ y=0, \\ z=0,\ z=36. \end{cases}$

4.6. $\iiint\limits_{V} y^2 z\cos(xyz)\ dx\,dy\,dz;$

$V \begin{cases} x=1,\ y=2\pi,\ z=2, \\ x=0,\ y=1,\ z=0. \end{cases}$

4.7.
$$\iiint\limits_V y^2\cos\left(\frac{\pi}{4}xy\right) dx\, dy\, dz;$$
$$V \begin{cases} x=0,\ y=-1,\ \text{y}=x/2, \\ z=0,\ z=-\pi^2. \end{cases}$$

4.8.
$$\iiint\limits_V x^2 z\sin\frac{xyz}{4} dx\, dy\, dz;$$
$$V \begin{cases} x=1,\ y=2\pi,\ \text{z}=4, \\ x=0,\ y=0,\ z=0. \end{cases}$$

4.9.
$$\iiint\limits_V y^2 e^{-xy} dx\, dy\, dz;$$
$$V \begin{cases} x=0,\ y=-2,\ \text{y}=4x, \\ z=0,\ z=1. \end{cases}$$

4.10.
$$\iiint\limits_V 2y^2 z\, e^{2xyz} dx\, dy\, dz;$$
$$V \begin{cases} x=1,\ y=1,\ z=1, \\ x=0,\ y=0,\ z=0. \end{cases}$$

4.11.
$$\iiint\limits_V y^2\mathrm{ch}(2xy) dx\, dy\, dz;$$
$$V \begin{cases} x=0,\ y=1,\ \text{y}=x, \\ z=0,\ z=8. \end{cases}$$

4.12.
$$\iiint\limits_V x^2 z\,\mathrm{sh}(xyz) dx\, dy\, dz;$$
$$V \begin{cases} x=2,\ y=1,\ z=1, \\ x=0,\ y=0,\ z=0. \end{cases}$$

4.13.
$$\iiint\limits_V y^2 e^{xy/2} dx\, dy\, dz;$$
$$V \begin{cases} x=0,\ y=2,\ \text{y}=2x, \\ z=0,\ z=-1. \end{cases}$$

4.14.
$$\iiint\limits_V y^2 z\cos\frac{xyz}{3} dx\, dy\, dz;$$
$$V \begin{cases} x=3,\ y=1,\ z=2\pi, \\ x=0,\ y=0,\ z=0. \end{cases}$$

4.15.
$$\iiint\limits_V y^2\cos\left(\frac{\pi xy}{2}\right) dx\, dy\, dz;$$
$$V \begin{cases} x=0,\ y=-1,\ \text{y}=x, \\ z=0,\ z=2\pi^2. \end{cases}$$

4.16.
$$\iiint\limits_V x^2 z\,\mathrm{sh}(xyz) dx\, dy\, dz;$$
$$V \begin{cases} x=1,\ y=-1,\ z=1, \\ x=0,\ y=0,\ z=0. \end{cases}$$

4.17.
$$\iiint\limits_V y^2\cos(\pi xy) dx\, dy\, dz;$$
$$V \begin{cases} x=0,\ y=1,\ \text{y}=2x, \\ z=0,\ z=\pi^2. \end{cases}$$

4.18.
$$\iiint\limits_V 2x^2 z\,\mathrm{sh}(2xyz) dx\, dy\, dz;$$
$$V \begin{cases} x=2,\ y=1/2,\ z=1/2, \\ x=0,\ y=0,\ z=0. \end{cases}$$

4.19.
$$\iiint\limits_V x^2\mathrm{sh}(2xy) dx\, dy\, dz;$$
$$V \begin{cases} x=-1,\ y=x,\ \text{y}=0, \\ z=0,\ z=8. \end{cases}$$

4.20.
$$\iiint\limits_V x^2 z\sin\frac{xyz}{2} dx\, dy\, dz;$$
$$V \begin{cases} x=1,\ y=4,\ z=\pi, \\ x=0,\ y=0,\ z=0. \end{cases}$$

4.21.
$$\iiint\limits_{V} y^2 \operatorname{ch}(xy)\, dx\, dy\, dz;$$
$$V \begin{cases} x = 0,\ y = -1,\ \mathrm{y} = x, \\ z = 0,\ z = 2. \end{cases}$$

4.22.
$$\iiint\limits_{V} x^2 z\, \operatorname{ch}(xyz)\, dx\, dy\, dz;$$
$$V \begin{cases} x = 1,\ y = 1,\ \mathrm{z} = 1, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$$

4.23.
$$\iiint\limits_{V} x^2 \cos\left(\frac{\pi}{2} xy\right)\, dx\, dy\, dz;$$
$$V \begin{cases} x = 2,\ y = x,\ \mathrm{y} = 0, \\ z = 0,\ z = \pi. \end{cases}$$

4.24.
$$\iiint\limits_{V} y^2 z \cos\frac{xyz}{9}\, dx\, dy\, dz;$$
$$V \begin{cases} x = 9,\ y = 1,\ \mathrm{z} = 2\pi, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$$

4.25.
$$\iiint\limits_{V} x^2 \cos(\pi xy)\, dx\, dy\, dz;$$
$$V \begin{cases} x = 1,\ y = 2x,\ \mathrm{y} = 0, \\ z = 0,\ z = 4\pi. \end{cases}$$

4.26.
$$\iiint\limits_{V} y^2 z\, \operatorname{ch}\left(\frac{xyz}{2}\right)\, dx\, dy\, dz;$$
$$V \begin{cases} x = 2,\ y = -1,\ \mathrm{z} = 2, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$$

4.27.
$$\iiint\limits_{V} y^2 \operatorname{ch}(3xy)\, dx\, dy\, dz;$$
$$V \begin{cases} x = 0,\ y = 2,\ \mathrm{y} = 6x, \\ z = 0,\ z = -3. \end{cases}$$

4.28.
$$\iiint\limits_{V} 2y^2 z\, \operatorname{ch}(2xyz)\, dx\, dy\, dz;$$
$$V \begin{cases} x = \dfrac{1}{2},\ y = 2,\ \mathrm{z} = -1, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$$

4.29.
$$\iiint\limits_{V} x^2 \sin(4\pi xy)\, dx\, dy\, dz;$$
$$V \begin{cases} x = 1,\ y = x/2,\ \mathrm{y} = 0, \\ z = 0,\ z = 8\pi. \end{cases}$$

4.30.
$$\iiint\limits_{V} 8y^2 z\, e^{-xyz}\, dx\, dy\, dz;$$
$$V \begin{cases} x = 2,\ y = -1,\ \mathrm{z} = 2, \\ x = 0,\ \mathrm{y} = 0,\ z = 0. \end{cases}$$

4.31.
$$\iiint\limits_{V} x^2 \operatorname{sh}(xy)\, dx\, dy\, dz;$$
$$V \begin{cases} x = 2,\ y = x/2,\ \mathrm{y} = 0, \\ z = 0,\ z = 1. \end{cases}$$

Задача 5. Вычислить.

$$\iiint\limits_V x\,dx\,dy\,dz;$$

5.1. $V:\ y=10x,\ y=0,\ x=1,$
$\quad z=xy,\ z=0.$

$$\iiint\limits_V 15\left(y^2+z^2\right)dx\,dy\,dz;$$

5.3. $V:\ z=x+y,\ x+y=1,$
$\quad x=0,\ y=0,\ z=0.$

$$\iiint\limits_V \left(1+2x^3\right)dx\,dy\,dz;$$

5.5. $V:\ y=9x,\ y=0,\ x=1,$
$\quad z=\sqrt{xy},\ z=0.$

$$\iiint\limits_V y\,dx\,dy\,dz;$$

5.7. $V:\ y=15x,\ y=0,\ x=1,$
$\quad z=xy,\ z=0.$

$$\iiint\limits_V \left(3x^2+y^2\right)dx\,dy\,dz;$$

5.9. $V:\ z=10y,\ x+y=1,$
$\quad x=0,\ y=0,\ z=0.$

$$\iiint\limits_V \left(4+8z^3\right)dx\,dy\,dz;$$

5.11. $V:\ y=x,\ y=0,\ x=1,$
$\quad z=\sqrt{xy},\ z=0.$

$$\iiint\limits_V \frac{dx\,dy\,dz}{\left(1+\dfrac{x}{3}+\dfrac{y}{4}+\dfrac{z}{8}\right)^4};$$

5.2. $V:\ 1+\dfrac{x}{3}+\dfrac{y}{4}+\dfrac{z}{8}=1,$
$\quad x=0,\ y=0,\ z=0.$

$$\iiint\limits_V \left(3x+4y\right)dx\,dy\,dz;$$

5.4. $V:\ y=x,\ y=0,\ x=1,$
$\quad z=5\left(x^2+y^2\right),\ z=0.$

$$\iiint\limits_V \left(27+54y^3\right)dx\,dy\,dz;$$

5.6. $V:\ y=x,\ y=0,\ x=1,$
$\quad z=\sqrt{xy},\ z=0.$

$$\iiint\limits_V \frac{dx\,dy\,dz}{\left(1+\dfrac{x}{16}+\dfrac{y}{8}+\dfrac{z}{3}\right)^5};$$

5.8. $V:\ \dfrac{x}{16}+\dfrac{y}{5}+\dfrac{z}{3}=1,$
$\quad x=0,\ y=0,\ z=0.$

$$\iiint\limits_V \left(15x+30z\right)dx\,dy\,dz;$$

5.10. $V:\ z=x^2+3y^2,\ z=0,$
$\quad y=x,\ y=0,\ z=0.$

$$\iiint\limits_V \left(1+2x^3\right)dx\,dy\,dz;$$

5.12. $V:\ y=36x,\ y=0,\ x=1,$
$\quad z=\sqrt{xy},\ z=0.$

$$\iiint\limits_V 21xz\ dx\ dy\ dz;$$

5.13. $V: y = x,\ y = 0,\ x = 2,$
$z = xy,\ z = 0.$

$$\iiint\limits_V \frac{dx\ dy\ dz}{\left(1 + \dfrac{x}{10} + \dfrac{y}{8} + \dfrac{z}{3}\right)^6};$$

5.14. $V: \dfrac{x}{10} + \dfrac{y}{8} + \dfrac{z}{3} = 1,$
$x = 0,\ y = 0,\ z = 0.$

$$\iiint\limits_V \left(x^2 + 3y^2\right) dx\ dy\ dz;$$

5.15. $V: z = 10x,\ x + y = 1,$
$x = 0,\ y = 0,\ z = 0.$

$$\iiint\limits_V \left(60y + 90z\right) dx\ dy\ dz;$$

5.16. $V: y = x,\ y = 0,\ x = 1,$
$z = x^2 + y^2,\ z = 0.$

$$\iiint\limits_V \left(\frac{10}{3}x + \frac{5}{3}\right) dx\ dy\ dz;$$

5.17. $V: y = 9x,\ y = 0,\ x = 1,$
$z = \sqrt{xy},\ z = 0.$

$$\iiint\limits_V \left(9 + 18z\right) dx\ dy\ dz;$$

5.18. $V: y = 4x,\ y = 0,\ x = 1,$
$z = \sqrt{xy},\ z = 0.$

$$\iiint\limits_V 3y^2\ dx\ dy\ dz;$$

5.19. $V: y = 2x,\ y = 0,\ x = 2,$
$z = xy,\ z = 0.$

$$\iiint\limits_V \frac{dx\ dy\ dz}{\left(1 + \dfrac{x}{2} + \dfrac{y}{4} + \dfrac{z}{6}\right)^6};$$

5.20. $V: \dfrac{x}{2} + \dfrac{y}{4} + \dfrac{z}{6} = 1,$
$x = 0,\ y = 0,\ z = 0.$

$$\iiint\limits_V x^2\ dx\ dy\ dz;$$

5.21. $V: z = 10(x + 3y),\ x + y = 1,$
$x = 0,\ y = 0,\ z = 0.$

$$\iiint\limits_V \left(8y + 12z\right) dx\ dy\ dz;$$

5.22. $V: y = x,\ y = 0,\ x = 1,$
$z = 3x^2 + 2y^2,\ z = 0.$

$$\iiint\limits_V 63\left(1 + 2\sqrt{y}\right) dx\ dy\ dz;$$

5.23. $V: y = x,\ y = 0,\ x = 1,$
$z = \sqrt{xy},\ z = 0.$

$$\iiint\limits_V \left(x + y\right) dx\ dy\ dz;$$

5.24. $V: y = x,\ y = 0,\ x = 1,$
$z = 30x^2 + 60y^2,\ z = 0.$

$$\iiint\limits_V \frac{dx\,dy\,dz}{\left(1+\dfrac{x}{6}+\dfrac{y}{4}+\dfrac{z}{16}\right)^6};$$

5.25. $V: \dfrac{x}{6}+\dfrac{y}{4}+\dfrac{z}{16}=1,$

$\quad\quad x=0,\ y=0,\ z=0.$

$$\iiint\limits_V xyz\,dx\,dy\,dz;$$

5.26. $V: y=x,\ y=0,\ x=2,$

$\quad\quad z=xy,\ z=0.$

$$\iiint\limits_V y^2\,dx\,dy\,dz;$$

5.27. $V: z=10(3x+y),\ \mathrm{x}+y=1,$

$\quad\quad x=0,\ y=0,\ z=0.$

$$\iiint\limits_V \left(5x+\frac{3z}{2}\right)dx\,dy\,dz;$$

5.28. $V: y=x,\ y=0,\ x=2,$

$\quad\quad z=x^2+15y^2,\ z=0.$

$$\iiint\limits_V \left(x^2+4y^2\right)dx\,dy\,dz;$$

5.29. $V: z=20(2x+y),\ \mathrm{x}+y=1,$

$\quad\quad x=0,\ y=0,\ z=0.$

$$\iiint\limits_V \frac{dx\,dy\,dz}{\left(1+\dfrac{x}{8}+\dfrac{y}{3}+\dfrac{z}{5}\right)^6};$$

5.30. $V: \dfrac{x}{8}+\dfrac{y}{3}+\dfrac{z}{5}=1,$

$\quad\quad x=0,\ y=0,\ z=0.$

$$\iiint\limits_V x^2z\,dx\,dy\,dz;$$

5.31. $V: y=3x,\ y=0,\ x=2,$

$\quad\quad z=xy,\ z=0.$

Задача 6. Найти площадь фигуры, ограниченной данными линиями.

6.1. $y=3/x,\ y=4e^x,\ y=3,\ y=4.$

6.2. $x=\sqrt{36-y^2},\ x=6-\sqrt{36-y^2}.$

6.3. $x^2+y^2=72,\ 6y=-x^2\ \left(y\le 0\right).$

6.4. $x=8-y^2,\ x=-2y.$

6.5. $y = \dfrac{3}{x}$, $y = 8e^x$, $y = 3$, $y = 8$.

6.6. $y = \dfrac{\sqrt{x}}{2}$, $y = \dfrac{1}{2x}$, $x = 16$.

6.7. $x = 5 - y^2$, $x = -4y$.

6.8. $x^2 + y^2 = 12$, $-\sqrt{6}y = x^2$ $(y \le 0)$.

6.9. $y = \sqrt{12 - x^2}$, $y = 2\sqrt{3} - \sqrt{12 - x^2}$, $x = 0$ $(x \ge 0)$.

6.10. $y = \dfrac{3}{2}\sqrt{x}$, $y = \dfrac{3}{2x}$, $x = 9$.

6.11. $y = \sqrt{24 - x^2}$, $2\sqrt{3}y = x^2$, $x = 0$ $(x \ge 0)$.

6.12. $y = \sin x$, $y = \cos x$, $x = 0$, $(x \ge 0)$.

6.13. $y = 20 - x^2$, $y = -8x$.

6.14. $y = \sqrt{18 - x^2}$, $y = 3\sqrt{2} - \sqrt{18 - x^2}$.

6.15. $y = 32 - x^2$, $y = -4x$.

6.16. $y = 2/x$, $y = 5e^x$, $y = 2$, $y = 5$.

6.17. $x^2 + y^2 = 36$, $3\sqrt{2}y = x^2$ $(y \ge 0)$.

6.18. $y = 3\sqrt{x}$, $y = 3/x$, $x = 4$.

6.19. $y = 6 - \sqrt{36 - x^2}$, $y = \sqrt{36 - x^2}$, $x = 0$ $(x \ge 0)$.

6.20. $y = 25/4 - x^2$, $y = x - 5/2$.

6.21. $y = \sqrt{x}$, $y = 1/x$, $x = 16$.

6.22. $y = 2/x,\ y = 7e^x,\ y = 2,\ y = 7.$

6.23. $x = 27 - y^2,\ x = -6y.$

6.24. $x = \sqrt{72 - y^2},\ 6x = y^2,\ y = 0\ \left(y \geq 0\right).$

6.25. $y = \sqrt{6 - x^2},\ y = \sqrt{6} - \sqrt{6 - x^2}.$

6.26. $y = \dfrac{3}{2}\sqrt{x},\ y = \dfrac{3}{2x},\ \mathrm{x} = 4.$

6.27. $y = \sin x,\ y = \cos x,\ \mathrm{x} = 0,\ \left(x \leq 0\right).$

6.28. $y = \dfrac{1}{x},\ y = 6e^x,\ y = 1,\ y = 6.$

6.29. $y = 3\sqrt{x},\ y = 3/x,\ x = 9.$

6.30. $y = 11 - x^2,\ y = -10x.$

6.31. $x^2 + y^2 = 12,\quad x\sqrt{6} = y^2\ \left(x \geq 0\right).$

Задача 7. Найти площадь фигуры, ограниченной данными линиями.

7.1. $\begin{aligned} & y^2 - 2y + x^2 = 0, \\ & y^2 - 4y + x^2 = 0, \\ & y = x/\sqrt{3},\ y = \sqrt{3}x. \end{aligned}$

7.2. $\begin{aligned} & x^2 - 4x + y^2 = 0, \\ & x^2 - 8x + y^2 = 0, \\ & y = 0,\ y = x/\sqrt{3}. \end{aligned}$

7.3. $\begin{aligned} & y^2 - 6y + x^2 = 0, \\ & y^2 - 8y + x^2 = 0, \\ & y = x/\sqrt{3},\ y = \sqrt{3}x. \end{aligned}$

7.4. $\begin{aligned} & x^2 - 2x + y^2 = 0, \\ & x^2 - 4x + y^2 = 0, \\ & y = 0,\ y = x. \end{aligned}$

$$y^2 - 8y + x^2 = 0,$$

7.5. $y^2 - 10y + x^2 = 0,$

$$y = x/\sqrt{3},\ y = \sqrt{3}x.$$

$$x^2 - 4x + y^2 = 0,$$

7.6. $x^2 - 8x + y^2 = 0,$

$$y = 0,\ y = x.$$

$$y^2 - 4y + x^2 = 0,$$

7.7. $y^2 - 6y + x^2 = 0,$

$$y = x,\ x = 0.$$

$$x^2 - 2x + y^2 = 0,$$

7.8. $x^2 - 10x + y^2 = 0,$

$$y = 0,\ y = \sqrt{3}x.$$

$$y^2 - 6y + x^2 = 0,$$

7.9. $y^2 - 10y + x^2 = 0,$

$$y = x,\ x = 0.$$

$$x^2 - 2x + y^2 = 0,$$

7.10. $x^2 - 4x + y^2 = 0,$

$$y = x/\sqrt{3},\ y = \sqrt{3}x.$$

$$y^2 - 2y + x^2 = 0,$$

7.11. $y^2 - 4y + x^2 = 0,$

$$y = \sqrt{3}x,\ x = 0.$$

$$x^2 - 2x + y^2 = 0,$$

7.12. $x^2 - 6x + y^2 = 0,$

$$y = x/\sqrt{3},\ y = \sqrt{3}x.$$

$$y^2 - 4y + x^2 = 0,$$

7.13. $y^2 - 6y + x^2 = 0,$

$$y = \sqrt{3}x,\ x = 0.$$

$$x^2 - 2x + y^2 = 0,$$

7.14. $x^2 - 8x + y^2 = 0,$

$$y = x/\sqrt{3},\ y = \sqrt{3}x.$$

$$y^2 - 2y + x^2 = 0,$$

7.15. $y^2 - 6y + x^2 = 0,$

$$y = x/\sqrt{3},\ y = 0.$$

$$x^2 - 2x + y^2 = 0,$$

7.16. $x^2 - 4x + y^2 = 0,$

$$y = 0,\ y = x/\sqrt{3}.$$

$$y^2 - 2y + x^2 = 0,$$

7.17. $y^2 - 10y + x^2 = 0,$

$$y = x/\sqrt{3},\ y = \sqrt{3}x.$$

$$x^2 - 2x + y^2 = 0,$$

7.18. $x^2 - 6x + y^2 = 0,$

$$y = 0,\ y = x/\sqrt{3}.$$

$$y^2 - 4y + x^2 = 0,$$

7.19. $y^2 - 10y + x^2 = 0,$

$$y = x/\sqrt{3},\ y = \sqrt{3}x.$$

$$x^2 - 2x + y^2 = 0,$$

7.20. $x^2 - 6x + y^2 = 0,$

$$y = 0,\ y = x.$$

7.21. $\begin{aligned} &y^2 - 2y + x^2 = 0, \\ &y^2 - 4y + x^2 = 0, \\ &y = x, \ x = 0. \end{aligned}$

7.22. $\begin{aligned} &x^2 - 2x + y^2 = 0, \\ &x^2 - 4x + y^2 = 0, \\ &y = 0, \ y = \sqrt{3}x. \end{aligned}$

7.23. $\begin{aligned} &y^2 - 6y + x^2 = 0, \\ &y^2 - 8y + x^2 = 0, \\ &y = x, \ x = 0. \end{aligned}$

7.24. $\begin{aligned} &x^2 - 4x + y^2 = 0, \\ &x^2 - 8x + y^2 = 0, \\ &y = 0, \ y = \sqrt{3}x. \end{aligned}$

7.25. $\begin{aligned} &y^2 - 4y + x^2 = 0, \\ &y^2 - 8y + x^2 = 0, \\ &y = x, \ x = 0. \end{aligned}$

7.26. $\begin{aligned} &x^2 - 4x + y^2 = 0, \\ &x^2 - 8x + y^2 = 0, \\ &y = x/\sqrt{3}, \ y = \sqrt{3}x. \end{aligned}$

7.27. $\begin{aligned} &y^2 - 4y + x^2 = 0, \\ &y^2 - 8y + x^2 = 0, \\ &y = \sqrt{3}x, \ x = 0. \end{aligned}$

7.28. $\begin{aligned} &x^2 - 4x + y^2 = 0, \\ &x^2 - 6x + y^2 = 0, \\ &y = x/\sqrt{3}, \ y = \sqrt{3}x. \end{aligned}$

7.29. $\begin{aligned} &y^2 - 2y + x^2 = 0, \\ &y^2 - 10y + x^2 = 0, \\ &y = x/\sqrt{3}, \ x = 0. \end{aligned}$

7.30. $\begin{aligned} &x^2 - 6x + y^2 = 0, \\ &x^2 - 10x + y^2 = 0, \\ &y = x/\sqrt{3}, \ y = \sqrt{3}x. \end{aligned}$

7.31. $\begin{aligned} &y^2 - 4y + x^2 = 0, \\ &y^2 - 8y + x^2 = 0, \\ &y = x/\sqrt{3}, \ x = 0. \end{aligned}$

Задача 8. Пластинка D задана ограничивающими ее кривыми, μ - поверхностная плотность. Найти массу пластинки.

8.1. $\begin{aligned} &D: x = 1, \ y = 0, \ y^2 = 4x \ \ (y \geq 0); \\ &\quad \mu = 7x^2 + y. \end{aligned}$

8.2. $\begin{aligned} &D: x^2 + y^2 = 1, \ x^2 + y^2 = 4, \\ &\quad x = 0, \ y = 0 \ (x \geq 0, \ y \geq 0); \\ &\quad \mu = (x + y)/(x^2 + y^2). \end{aligned}$

8.3. $D:\ x=1,\ y=0,\ y^2=4x\ \ (y\ge 0);$
$\mu=7x^2/2+5y.$

8.4. $D:\ x^2+y^2=9,\ x^2+y^2=16,$
$x=0,\ y=0\ (x\ge 0,\ y\ge 0);$
$\mu=(2x+5y)/(x^2+y^2).$

8.5. $D:\ x=2,\ y=0,\ y^2=2x\ \ (y\ge 0);$
$\mu=7x^2/8+2y.$

8.6. $D:\ x^2+y^2=1,\ x^2+y^2=16,$
$x=0,\ y=0\ (x\ge 0,\ y\ge 0);$
$\mu=(x+y)/(x^2+y^2).$

8.7. $D:\ x=2,\ y=0,\ y^2=x/2\ \ (y\ge 0);$
$\mu=7x^2/2+6y.$

8.8. $D:\ x^2+y^2=4,\ x^2+y^2=25,$
$x=0,\ y=0\ (x\ge 0,\ y\le 0);$
$\mu=(2x-3y)/(x^2+y^2).$

8.9. $D:\ x=1,\ y=0,\ y^2=4x\ \ (y\ge 0);$
$\mu=x+3y^2.$

8.10. $D:\ x^2+y^2=1,\ x^2+y^2=9,$
$x=0,\ y=0\ (x\ge 0,\ y\le 0);$
$\mu=(x-y)/(x^2+y^2).$

8.11. $D:\ x=1,\ y=0,\ y^2=x\ \ (y\ge 0);$
$\mu=3x+6y^2.$

8.12. $D:\ x^2+y^2=9,\ x^2+y^2=25,$
$x=0,\ y=0\ (x\le 0,\ y\ge 0);$
$\mu=(2y-x)/(x^2+y^2).$

8.13. $D:\ x=2,\ y=0,\ y^2=x/2\ \ (y\ge 0);$
$\mu=2x+3y^2.$

8.14. $D:\ x^2+y^2=4,\ x^2+y^2=16,$
$x=0,\ y=0\ (x\le 0,\ y\ge 0);$
$\mu=(2y-3x)/(x^2+y^2).$

8.15. $D:\ x=\dfrac{1}{2},\ y=0,\ y^2=8x\ \ (y\ge 0);$
$\mu=7x+3y^2.$

8.16. $D:\ x^2+y^2=9,\ x^2+y^2=16,$
$x=0,\ y=0\ (x\le 0,\ y\ge 0);$
$\mu=(2y-5x)/(x^2+y^2).$

8.17. $D:\ x=1,\ y=0,\ y^2=4x\ \ (y\ge 0);$
$\mu=7x^2+2y.$

8.18. $D:\ x^2+y^2=1,\ x^2+y^2=16,$
$x=0,\ y=0\ (x\ge 0,\ y\ge 0);$
$\mu=(x+3y)/(x^2+y^2).$

8.19. $D: x = 2, y^2 = 2x, y = 0 \ (y \geq 0);$
$\mu = 7x^2/4 + y/2.$

8.20. $D: x^2 + y^2 = 1, \ x^2 + y^2 = 4,$
$x = 0, y = 0 \ (x \geq 0, y \geq 0);$
$\mu = (x + 2y)/(x^2 + y^2).$

8.21. $D: x = 2, y = 0, y^2 = 2x \ (y \geq 0);$
$\mu = 7x^2/4 + y.$

8.22. $D: x^2 + y^2 = 1, \ x^2 + y^2 = 9,$
$x = 0, y = 0 \ (x \geq 0, y \leq 0);$
$\mu = (2x - y)/(x^2 + y^2).$

8.23. $D: x = 2, y = 0, y^2 = x/2 \ (y \geq 0);$
$\mu = 7x^2/2 + 8y.$

8.24. $D: x^2 + y^2 = 1, \ x^2 + y^2 = 25,$
$x = 0, y = 0 \ (x \geq 0, y \leq 0);$
$\mu = (x - 4y)/(x^2 + y^2).$

8.25. $D: x = 1, y = 0, y^2 = 4x \ (y \geq 0);$
$\mu = 6x + 3y^2.$

8.26. $D: x^2 + y^2 = 4, \ x^2 + y^2 = 16,$
$x = 0, y = 0 \ (x \geq 0, y \leq 0);$
$\mu = (3x - y)/(x^2 + y^2).$

8.27. $D: x = 2, y = 0, y^2 = x/2 \ (y \geq 0);$
$\mu = 4x + 6y^2.$

8.28. $D: x^2 + y^2 = 4, \ x^2 + y^2 = 9,$
$x = 0, y = 0 \ (x \leq 0, y \geq 0);$
$\mu = (y - 4x)/(x^2 + y^2).$

8.29. $D: x = \dfrac{1}{2}, y = 0, y^2 = 2x \ (y \geq 0);$
$\mu = 4x + 9y^2.$

8.30. $D: x^2 + y^2 = 4, \ x^2 + y^2 = 9,$
$x = 0, y = 0 \ (x \leq 0, y \geq 0);$
$\mu = (y - 2x)/(x^2 + y^2).$

8.31. $D: x = \dfrac{1}{4}, y = 0, y^2 = 16x \ (y \geq 0);$
$\mu = 16x + 9y^2/2.$

Задача 9. Пластинка D задана неравенствами, μ - поверхностная плотность. Найти массу пластинки.

9.1. $D:\ x^2 + y^2/4 \leq 1;$
$\mu = y^2.$

9.2. $D:\ 1 \leq x^2/9 + y^2/4 \leq 2;$
$y \geq 0,\ y \leq \dfrac{2}{3}x;$
$\mu = y/x.$

9.3. $D:\ x^2/9 + y^2/25 \leq 1;$
$y \geq 0;$
$\mu = x^2 y.$

9.4. $D:\ x^2/9 + y^2/25 \leq 1;$
$y \geq 0;$
$\mu = 7x^2 y/18.$

9.5. $D:\ 1 \leq x^2/9 + y^2/4 \leq 4;$
$y \geq 0,\ y \leq x/2;$
$\mu = 8\,y/x^3.$

9.6. $D:\ x^2/9 + y^2 \leq 1;$
$x \geq 0;$
$\mu = 7xy^6.$

9.7. $D:\ x^2/4 + y^2 \leq 1;$
$\mu = 4y^4.$

9.8. $D:\ 1 \leq x^2/4 + y^2/9 \leq 4;$
$x \geq 0,\ y \leq 3x/2;$
$\mu = x/y.$

9.9. $D:\ 1 \leq x^2/16 + y^2/4 \leq 4;$
$x \geq 0,\ y \leq x/2;$
$\mu = x/y.$

9.10. $D:\ x^2/4 + y^2/9 \leq 1;$
$x \geq 0,\ y \geq 0;$
$\mu = x^3 y.$

9.11. $D:\ x^2/4 + y^2 \leq 1;$
$x \geq 0,\ y \geq 0;$
$\mu = 6x^3 y^3.$

9.12. $D:\ 1 \leq x^2/4 + y^2 \leq 25;$
$x \geq 0,\ y \leq x/2;$
$\mu = x/y^3.$

9.13. $D:\ x^2/9 + y^2/4 \leq 1;$
$\mu = x^2 y^2.$

9.14. $D:\ x^2/16 + y^2 \leq 1;$
$x \geq 0,\ y \geq 0;$
$\mu = 5xy^7.$

9.15. $D:\ x^2/4 + y^2 \leq 1;$
$x \geq 0,\ y \geq 0;$
$\mu = 30x^3 y^7.$

9.16. $D:\ 1 \leq x^2/9 + y^2/4 \leq 3;$
$y \geq 0,\ y \leq \dfrac{2}{3}x;$
$\mu = y/x.$

9.17.
$$D:\ x^2 + y^2/25 \le 1;$$
$$y \ge 0;$$
$$\mu = 7x^4 y.$$

9.18.
$$D:\ x^2 + y^2/9 \le 1;$$
$$y \ge 0;$$
$$\mu = 35x^4 y^3.$$

9.19.
$$D:\ x^2/4 + y^2/9 \le 1;$$
$$\mu = x^2.$$

9.20.
$$D:\ 1 \le x^2 + y^2/16 \le 9;$$
$$y \ge 0,\ y \le 4x;$$
$$\mu = y/x^3.$$

9.21.
$$D:\ x^2/9 + y^2 \le 1;$$
$$x \ge 0;$$
$$\mu = 11xy^8.$$

9.22.
$$D:\ 1 \le x^2/4 + y^2/16 \le 5;$$
$$x \ge 0,\ y \le 2x;$$
$$\mu = x/y.$$

9.23.
$$D:\ 1 \le x^2/9 + y^2/4 \le 5;$$
$$x \ge 0,\ y \le 2x/3;$$
$$\mu = x/y.$$

9.24.
$$D:\ x^2/4 + y^2/9 \le 1;$$
$$x \ge 0,\ y \ge 0;$$
$$\mu = x^5 y.$$

9.25.
$$D:\ x^2/4 + y^2/25 \le 1;$$
$$\mu = x^4.$$

9.26.
$$D:\ x^2 + y^2/4 \le 1;$$
$$x \ge 0,\ y \ge 0;$$
$$\mu = 15x^5 y^3.$$

9.27.
$$D:\ 1 \le x^2/4 + y^2/9 \le 36;$$
$$x \ge 0,\ y \ge \frac{3}{2}x;$$
$$\mu = 9x/y^3.$$

9.28.
$$D:\ x^2/100 + y^2 \le 1;$$
$$x \ge 0,\ y \ge 0;$$
$$\mu = 6xy^9.$$

9.29.
$$D:\ x^2/16 + y^2 \le 1;$$
$$x \ge 0,\ y \ge 0;$$
$$\mu = 105x^3 y^9.$$

9.30.
$$D:\ 1 \le x^2/9 + y^2/16 \le 2;$$
$$y \ge 0,\ y \le \frac{4}{3}x;$$
$$\mu = 27y/x^5.$$

9.31.
$$D:\ 1 \le x^2/16 + y^2 \le 3;$$
$$x \ge 0,\ y \ge x/4;$$
$$\mu = x/y^5.$$

Задача 10. Найти объем тела, заданного ограничивающими его поверхностями.

10.1. $y = 16\sqrt{2x}, \ y = \sqrt{2x},$
$z = 0, \ x + z = 2.$

10.2. $y = 5\sqrt{x}, \ y = 5x/3,$
$z = 0, \ z = 5 + 5\sqrt{x}/3.$

10.3. $x^2 + y^2 = 2, \ y = \sqrt{x}, \ y = 0,$
$z = 0, \ z = 15x.$

10.4. $x + y = 2, \ y = \sqrt{x},$
$z = 12y, \ z = 0.$

10.5. $x = 20\sqrt{2y}, \ x = 5\sqrt{2y},$
$z = 0, \ z + y = 1/2.$

10.6. $x = 5\sqrt{y}/2, \ x = 5y/6,$
$z = 0, \ z = \dfrac{5}{6}\left(3 + \sqrt{y}\right).$

10.7. $x^2 + y^2 = 2, \ x = \sqrt{y}, \ x = 0,$
$z = 0, \ z = 30y.$

10.8. $x + y = 2, \ x = \sqrt{y},$
$z = 12x/5, \ z = 0.$

10.9. $y = 17\sqrt{2x}, \ y = 2\sqrt{2x},$
$z = 0, \ x + z = 1/2.$

10.10. $y = 5\sqrt{x}/3, \ y = 5x/9,$
$z = 0, \ z = 5\left(3 + \sqrt{x}\right)/9.$

10.11. $x^2 + y^2 = 8, \ y = \sqrt{2x}, \ y = 0,$
$z = 0, \ z = 15x/11.$

10.12. $x + y = 4, \ y = \sqrt{2x},$
$z = 3y, \ z = 0.$

10.13. $x = \dfrac{5}{6}\sqrt{y}, \ x = \dfrac{5}{18}y,$
$z = 0, \ z = \dfrac{5}{18}\left(3 + \sqrt{y}\right).$

10.14. $x = 19\sqrt{2y}, \ x = 4\sqrt{2y},$
$z = 0, \ z + y = 2.$

10.15. $x^2 + y^2 = 8, \ x = \sqrt{2y}, \ x = 0,$
$z = 30y/11, \ z = 0.$

10.16. $x + y = 4, \ x = \sqrt{2y},$
$z = 3x/5, \ z = 0.$

10.17. $y = 6\sqrt{3x}, \ y = \sqrt{3x},$
$z = 0, \ x + z = 3.$

10.18. $y = \dfrac{5}{6}\sqrt{x}, \ y = \dfrac{5}{18}x,$
$z = 0, \ z = \dfrac{5}{18}\left(3 + \sqrt{x}\right).$

10.19. $x^2 + y^2 = 18,\ y = \sqrt{3x},\ y = 0,$
$z = 0,\ z = 5x/11.$

10.20. $x + y = 6,\ y = \sqrt{3x},$
$z = 4y,\ z = 0.$

10.21. $x = 7\sqrt{3y},\ x = 2\sqrt{3y},$
$z = 0,\ z + y = 3.$

10.22. $x = 5\sqrt{y}\big/3,\ x = 5y/9,$
$z = 0,\ z = 5\big(3 + \sqrt{y}\big)\big/9.$

10.23. $x^2 + y^2 = 18,\ x = \sqrt{3y},\ x = 0,$
$z = 0,\ z = 10y/11.$

10.24. $x + y = 6,\ x = \sqrt{3y},$
$z = 4x/5,\ z = 0.$

10.25. $y = \sqrt{15x},\ y = \sqrt{15x},$
$z = 0,\ z = \sqrt{15}\big(1 + \sqrt{x}\big).$

10.26. $x^2 + y^2 = 50,\ y = \sqrt{5x},$
$y = 0,\ z = 0,\ z = 3x/11.$

10.27. $x + y = 8,\ y = \sqrt{4x},$
$z = 3y,\ z = 0.$

10.28. $x = 16\sqrt{2y},\ x = \sqrt{2y},$
$z + y = 2,\ z = 0.$

10.29. $x = \sqrt{y},\ x = 15y,$
$z = 0,\ z = 15\big(1 + \sqrt{y}\big).$

10.30. $x^2 + y^2 = 50,\ x = \sqrt{5y},$
$x = 0,\ z = 0,\ z = 6y/11.$

10.31. $x = 17\sqrt{2y},\ x = 2\sqrt{2y},$
$z = 0,\ z + y = 1/2.$

Задача 11. Найти объем тела, заданного ограничивающими его поверхностями.

11.1. $x^2 + y^2 = 2y,$
$z = 5/4 - x^2,\ z = 0.$

11.2. $x^2 + y^2 = y,\ x^2 + y^2 = 4y,$
$z = \sqrt{x^2 + y^2},\ z = 0.$

11.3. $x^2 + y^2 = 8\sqrt{2}x,$
$z = x^2 + y^2 - 64,$
$z = 0\ \ \big(z \geq 0\big).$

11.4. $x^2 + y^2 + 4x = 0,$
$z = 8 - y^2,\ z = 0.$

11.5. $x^2 + y^2 = 6x, \ x^2 + y^2 = 9x,$
$z = \sqrt{x^2 + y^2}, \ z = 0,$
$y = 0 \ (y \le 0)$

11.6. $x^2 + y^2 = 6\sqrt{2}\,y,$
$z = x^2 + y^2 - 36,$
$z = 0 \ (z \ge 0).$

11.7. $x^2 + y^2 = 2y,$
$z = 9/4 - x^2, \ z = 0.$

11.8. $x^2 + y^2 = 2y, \ x^2 + y^2 = 5y,$
$z = \sqrt{x^2 + y^2}, \ z = 0.$

11.9. $x^2 + y^2 + 2\sqrt{2}\,y = 0,$
$z = x^2 + y^2 - 4,$
$z = 0 \ (z \ge 0).$

11.10. $x^2 + y^2 = 4x,$
$z = 10 - y^2, \ z = 0.$

11.11. $x^2 + y^2 = 7x, \ x^2 + y^2 = 9x,$
$z = \sqrt{x^2 + y^2}, \ z = 0,$
$y = 0 \ (y \le 0)$

11.12. $x^2 + y^2 = 8\sqrt{2}\,y,$
$z = x^2 + y^2 - 64,$
$z = 0 \ (z \ge 0).$

11.13. $x^2 + y^2 = 2y,$
$z = 13/4 - x^2, \ z = 0.$

11.14. $x^2 + y^2 = 3y, \ x^2 + y^2 = 6y,$
$z = \sqrt{x^2 + y^2}, \ z = 0.$

11.15. $x^2 + y^2 = 6\sqrt{2}\,x,$
$z = x^2 + y^2 - 36,$
$z = 0 \ (z \ge 0).$

11.16. $x^2 + y^2 = 2\sqrt{2}\,y,$
$z = x^2 + y^2 - 4,$
$z = 0 \ (z \ge 0).$

11.17. $x^2 + y^2 = 4x,$
$z = 12 - y^2, \ z = 0.$

11.18. $x^2 + y^2 = 8x, \ x^2 + y^2 = 11x,$
$z = \sqrt{x^2 + y^2}, \ z = 0,$
$y = 0 \ (y \le 0)$

11.19. $x^2 + y^2 = 4\sqrt{2}\,x,$
$z = x^2 + y^2 - 16,$
$z = 0 \ (z \ge 0).$

11.20. $x^2 + y^2 = 4y,$
$z = 4 - x^2, \ z = 0.$

11.21. $x^2 + y^2 = 4y, \ x^2 + y^2 = 7y,$
$z = \sqrt{x^2 + y^2}, \ z = 0.$

11.22. $x^2 + y^2 = 4\sqrt{2}y,$
$z = x^2 + y^2 - 16,$
$z = 0 \ \left(z \geq 0 \right).$

11.23. $x^2 + y^2 + 2x = 0,$
$z = 17/4 - y^2, \ z = 0.$

11.24. $x^2 + y^2 = 9x, \ x^2 + y^2 = 12x,$
$z = \sqrt{x^2 + y^2}, \ z = 0,$
$y = 0 \ \left(y \geq 0 \right)$

11.25. $x^2 + y^2 + 2\sqrt{2}x = 0,$
$z = x^2 + y^2 - 4,$
$z = 0 \ \left(z \geq 0 \right).$

11.26. $x^2 + y^2 = 4y,$
$z = 6 - x^2, \ z = 0.$

11.27. $x^2 + y^2 = 10x, \ x^2 + y^2 = 13x,$
$z = \sqrt{x^2 + y^2}, \ z = 0,$
$y = 0 \ \left(y \geq 0 \right)$

11.28. $x^2 + y^2 = 2\sqrt{2}x,$
$z = x^2 + y^2 - 4,$
$z = 0 \ \left(z \geq 0 \right).$

11.29. $x^2 + y^2 = 2x,$
$z = 21/4 - y^2, \ z = 0.$

11.30. $x^2 + y^2 = 5y, \ x^2 + y^2 = 8y,$
$z = \sqrt{x^2 + y^2}, \ z = 0.$

11.31. $x^2 + y^2 + 2x = 0,$
$z = 25/4 - y^2, \ z = 0.$

Задача 12. Найти объем тела, заданного ограничивающими его поверхностями.

12.1. $y = 5x^2 + 2, \ y = 7,$
$z = 3y^2 - 7x^2 - 2,$
$z = 3y^2 - 7x^2 - 5.$

12.2. $y = 5x^2 - 2, \ y = -4x^2 + 7,$
$z = 4 + 9x^2 + 5y^2,$
$z = -1 + 9x^2 + 5y^2.$

12.3. $x = -5y^2 + 2, \ x = -3,$
$z = 3x^2 + y^2 + 1,$
$z = 3x^2 + y^2 - 5.$

12.4. $x = 2y^2 - 3, \ x = -7y^2 + 6,$
$z = 1 + \sqrt{x^2 + 16y^2},$
$z = -3 + \sqrt{x^2 + 16y^2}.$

$$y = -6x^2 + 8, \quad y = 2,$$
12.5. $z = x - x^2 - y^2 - 1,$
$$z = x - x^2 - y^2 - 5.$$

$$y = 5x^2 - 1, \quad y = -3x^2 + 1,$$
12.6. $z = -2 + \sqrt{3x^2 + y^2},$
$$z = -5 + \sqrt{3x^2 + y^2}.$$

$$x = 5y^2 - 9, \quad x = -4,$$
12.7. $z = x^2 + 4x - y^2 - 4,$
$$z = x^2 + 4x - y^2 + 2.$$

$$y = 6x^2 - 1, \quad y = 5,$$
12.8. $z = 2x^2 + x - y^2,$
$$z = 2x^2 + x - y^2 + 4.$$

$$x = 5y^2 - 1, \quad x = -3y^2 + 1,$$
12.9. $z = 2 - \sqrt{x^2 + 6y^2},$
$$z = -1 - \sqrt{x^2 + 6y^2}.$$

$$x = -3y^2 + 7, \quad x = 4,$$
12.10. $z = 2 + \sqrt{6x^2 + y^2},$
$$z = 3 + \sqrt{6x^2 + y^2}.$$

$$y = -5x^2 + 3, \quad y = -2,$$
12.11. $z = 2x^2 - 3y - 6y^2 - 1,$
$$z = 2x^2 - 3y - 6y^2 + 2.$$

$$y = x^2 - 5, \quad y = -x^2 + 3,$$
12.12. $z = 4 + \sqrt{5x^2 + 8y^2},$
$$z = 1 + \sqrt{5x^2 + 8y^2}.$$

$$x = 3y^2 - 5, \quad x = -2,$$
12.13. $z = 2 - \sqrt{x^2 + 16y^2},$
$$z = 8 - \sqrt{x^2 + 16y^2}.$$

$$x = y^2 - 2, \quad x = -4y^2 + 3,$$
12.14. $z = \sqrt{16 - x^2 - y^2} + 2,$
$$z = \sqrt{16 - x^2 - y^2} - 1.$$

$$y = 2x^2 - 1, \quad y = 1,$$
12.15. $z = x^2 - 5y^2 - 3,$
$$z = x^2 - 5y^2 - 6.$$

$$y = x^2 - 2, \quad y = -4x^2 + 3,$$
12.16. $z = 2 + \sqrt{x^2 + y^2},$
$$z = -1 + \sqrt{x^2 + y^2}.$$

$$x = -4y^2 + 1, \quad x = -3,$$
12.17. $z = x^2 - 7y^2 - 1,$
$$z = x^2 - 7y^2 + 2.$$

$$x = 7y^2 - 6, \quad x = -2y^2 + 3,$$
12.18. $z = 3 - 12y^2 + 5x^2,$
$$z = -2 - 12y^2 + 5x^2.$$

$$y = 1 - 2x^2, \quad y = -1,$$
12.19. $z = x^2 + 2y + y^2 - 2,$
$$z = x^2 + 2y + y^2 + 1.$$

$$y = x^2 - 7, \quad y = -8x^2 + 2,$$
12.20. $z = 3 - 12y^2 + 5x^2,$
$$z = -2 - 12y^2 + 5x^2.$$

12.21. $x = 2y^2 + 3, \ x = 5,$
$z = 1 + \sqrt{9x^2 + 4y^2},$
$z = 4 + \sqrt{9x^2 + 4y^2}.$

12.22. $y = 3x^2 + 4, \ y = 7,$
$z = 5 - \sqrt{2x^2 + 3y^2},$
$z = 1 - \sqrt{2x^2 + 3y^2}.$

12.23. $x = 5y^2 - 2, \ x = -4y^2 + 7,$
$z = 4 - \sqrt{2x^2 + 3y^2},$
$z = -1 - \sqrt{2x^2 + 3y^2}.$

12.24. $x = -2y^2 + 5, \ x = 3,$
$z = 5 - \sqrt{x^2 + 25y^2},$
$z = 2 - \sqrt{x^2 + 25y^2}.$

12.25. $y = -3x^2 + 5, \ y = 2,$
$z = 3 + \sqrt{5x^2 + y^2},$
$z = -1 + \sqrt{5x^2 + y^2}.$

12.26. $y = 3x^2 - 5, \ y = -6x^2 + 4,$
$z = 2 + 10x^2 - y^2,$
$z = -2 + 10y^2 - y^2.$

12.27. $x = 4y^2 + 2, \ x = 6,$
$z = x^2 + 4y^2 + y + 1,$
$z = x^2 + 4y^2 + y + 4.$

12.28. $x = 3y^2 - 2, \ x = -4y^2 + 5,$
$z = 4 - 7x^2 - 9y^2,$
$z = 1 - 7x^2 - 9y^2.$

12.29. $y = 2x^2 - 5, \ y = -3,$
$z = 2 + \sqrt{x^2 + 4y^2},$
$z = -1 + \sqrt{x^2 + 4y^2}.$

12.30. $y = 2x^2 - 3, \ y = -7x^2 + 6,$
$z = 1 - 5x^2 - 6y^2,$
$z = -3 - 5x^2 - 6y^2.$

12.31. $y = -2x^2 + 7, \ y = 5,$
$z = 1 - 2x^2 + 3y^2,$
$z = 4 - 2x^2 + 3y^2.$

Задача 13. Найти объем тела, заданного ограничивающими его поверхностями.

13.1. $z = \sqrt{9 - x^2 - y^2},$
$9z/2 = x^2 + y^2.$

13.2. $z = 15\sqrt{x^2 + y^2}\big/2,$
$z = 17/2 - x^2 - y^2.$

13.3. $z = \sqrt{4 - x^2 - y^2},$
$z = \sqrt{(x^2 + y^2)/255}.$

13.4. $z = \sqrt{64 - x^2 - y^2},\ z = 1,$
$x^2 + y^2 = 60$
(внутри цилиндра).

13.5. $z = \sqrt{\dfrac{16}{9} - x^2 - y^2},$
$2z = x^2 + y^2.$

13.6. $z = 3\sqrt{x^2 + y^2},$
$z = 10 - x^2 - y^2.$

13.7. $z = \sqrt{25 - x^2 - y^2},$
$z = \sqrt{(x^2 + y^2)/99}.$

13.8. $z = \sqrt{100 - x^2 - y^2},\ z = 6,$
$x^2 + y^2 = 51$
(внутри цилиндра).

13.9. $z = 21\sqrt{x^2 + y^2}/2,$
$z = 23/2 - x^2 - y^2.$

13.10. $z = \sqrt{16 - x^2 - y^2},$
$6z = x^2 + y^2.$

13.11. $z = \sqrt{9 - x^2 - y^2},$
$z = \sqrt{(x^2 + y^2)/80}.$

13.12. $z = \sqrt{81 - x^2 - y^2},\ z = 5,$
$x^2 + y^2 = 45$
(внутри цилиндра).

13.13. $z = \sqrt{1 - x^2 - y^2},$
$3z/2 = x^2 + y^2.$

13.14. $z = 6\sqrt{x^2 + y^2},$
$z = 16 - x^2 - y^2.$

13.15. $z = \sqrt{36 - x^2 - y^2},$
$z = \sqrt{(x^2 + y^2)/63}.$

13.16. $z = \sqrt{64 - x^2 - y^2},\ z = 4,$
$x^2 + y^2 = 39$
(внутри цилиндра).

13.17. $z = \sqrt{144 - x^2 - y^2},$
$18z = x^2 + y^2.$

13.18. $z = 3\sqrt{x^2 + y^2}/2,$
$z = 5/2 - x^2 - y^2.$

13.19. $z = \sqrt{9 - x^2 - y^2},$
$z = \sqrt{(x^2 + y^2)/35}.$

13.20. $z = \sqrt{49 - x^2 - y^2},\ z = 3,$
$x^2 + y^2 = 33$
(внутри цилиндра).

13.21. $z = \sqrt{36 - x^2 - y^2}$,
$9z = x^2 + y^2$.

13.22. $z = 9\sqrt{x^2 + y^2}$,
$z = 22 - x^2 - y^2$.

13.23. $z = \sqrt{16 - x^2 - y^2}$,
$z = \sqrt{(x^2 + y^2)/15}$.

13.24. $z = \sqrt{36 - x^2 - y^2}$, $z = 2$,
$x^2 + y^2 = 27$
(внутри цилиндра).

13.25. $z = \sqrt{4/9 - x^2 - y^2}$,
$z = x^2 + y^2$.

13.26. $z = 12\sqrt{x^2 + y^2}$,
$z = 28 - x^2 - y^2$.

13.27. $z = \sqrt{9 - x^2 - y^2}$,
$z = \sqrt{(x^2 + y^2)/8}$.

13.28. $z = \sqrt{25 - x^2 - y^2}$, $z = 1$,
$x^2 + y^2 = 21$
(внутри цилиндра).

13.29. $z = \sqrt{64 - x^2 - y^2}$,
$12z = x^2 + y^2$.

13.30. $z = 9\sqrt{x^2 + y^2}/2$,
$z = 11/2 - x^2 - y^2$.

13.31. $z = \sqrt{36 - x^2 - y^2}$,
$z = \sqrt{(x^2 + y^2)/3}$.

Задача 14. Найти объем тела, заданного ограничивающими его поверхностями.

14.1. $z = 2 - 12(x^2 + y^2)$,
$z = 24x + 2$.

14.2. $z = 10[(x-1)^2 + y^2] + 1$,
$z = 21 - 20x$.

14.3. $z = 8(x^2 + y^2) + 3$,
$z = 16x + 3$.

14.4. $z = 2 - 20[(x+1)^2 + y^2]$,
$z = -40 - 38x$.

14.5. $z = 4 - 14(x^2 + y^2)$,
$z = 4 - 28x$.

14.6. $z = 28[(x+1)^2 + y^2] + 3$,
$z = 56x + 59$.

14.7. $z = 32(x^2 + y^2) + 3,$
$z = 3 - 64x.$

14.8. $z = 4 - 6[(x-1)^2 + y^2],$
$z = 12x - 8.$

14.9. $z = 2 - 4(x^2 + y^2),$
$z = 8x + 2.$

14.10. $z = 22[(x-1)^2 + y^2] + 3,$
$z = 47 - 44x.$

14.11. $z = 24(x^2 + y^2) + 1,$
$z = 48x + 1.$

14.12. $z = 2 - 18[(x+1)^2 + y^2],$
$z = -36x - 34.$

14.13. $z = -16(x^2 + y^2) - 1,$
$z = -32x - 1.$

14.14. $z = 30[(x+1)^2 + y^2] + 1,$
$z = 60x + 61.$

14.15. $z = 26(x^2 + y^2) - 2,$
$z = -52x - 2.$

14.16. $z = -2[(x-1)^2 + y^2] - 1,$
$z = 4x - 5.$

14.17. $z = -2(x^2 + y^2) - 1,$
$z = 4y - 1.$

14.18. $z = 26[(x-1)^2 + y^2] - 2,$
$z = 50 - 52x.$

14.19. $z = 30(x^2 + y^2) + 1,$
$z = 60y + 1.$

14.20. $z = -16[(x+1)^2 + y^2] - 1,$
$z = -32x - 33.$

14.21. $z = 2 - 18(x^2 + y^2),$
$z = 2 - 36y.$

14.22. $z = 24[(x+1)^2 + y^2] + 1,$
$z = 48x + 49.$

14.23. $z = 22(x^2 + y^2) + 3,$
$z = 3 - 44y.$

14.24. $z = 2 - 4[(x-1)^2 + y^2],$
$z = 8x - 6.$

14.25. $z = 4 - 6(x^2 + y^2),$
$z = 12y + 4.$

14.26. $z = 32[(x-1)^2 + y^2] + 3,$
$z = 67 - 64x.$

14.27. $z = 28(x^2 + y^2) + 3,$
$z = 56y + 3.$

14.28. $z = 4 - 14[(x+1)^2 + y^2],$
$z = -28x - 24.$

14.29. $z = 2 - 20\left(x^2 + y^2\right),$
$z = 2 - 40y.$

14.30. $z = 8\left[(x+1)^2 + y^2\right] + 3,$
$z = 16x + 19.$

14.31. $z = 10\left(x^2 + y^2\right) + 1,$
$z = 1 - 20y.$

Задача 15. Найти объем тела, заданного неравенствами.

15.1. $1 \le x^2 + y^2 + z^2 \le 49,$
$-\sqrt{\dfrac{x^2 + y^2}{35}} \le z \le \sqrt{\dfrac{x^2 + y^2}{3}},$
$-x \le y \le 0.$

15.2. $1 \le x^2 + y^2 + z^2 \le 64,$
$\sqrt{\dfrac{x^2 + y^2}{15}} \le z \le \sqrt{\dfrac{x^2 + y^2}{3}},$
$-\sqrt{3}x \le y \le 0.$

15.3. $4 \le x^2 + y^2 + z^2 \le 64,$
$z \le \sqrt{\dfrac{x^2 + y^2}{3}}, \quad -\dfrac{x}{\sqrt{3}} \le y \le 0.$

15.4. $4 \le x^2 + y^2 + z^2 \le 36,$
$z \ge -\sqrt{\dfrac{x^2 + y^2}{63}}, \quad 0 \le y \le -\dfrac{x}{\sqrt{3}}.$

15.5. $1 \le x^2 + y^2 + z^2 \le 36,$
$z \ge \sqrt{\dfrac{x^2 + y^2}{99}}, \quad -\sqrt{3}x \le y \le \sqrt{3}x.$

15.6. $25 \le x^2 + y^2 + z^2 \le 100,$
$z \le -\sqrt{\dfrac{x^2 + y^2}{99}}, \quad \sqrt{3}x \le y \le -\sqrt{3}x.$

15.7. $1 \le x^2 + y^2 + z^2 \le 49,$
$0 \le z \le \sqrt{\dfrac{x^2 + y^2}{24}},$
$y \le -\dfrac{x}{\sqrt{3}}, \quad y \le -\sqrt{3}x.$

15.8. $25 \le x^2 + y^2 + z^2 \le 49,$
$-\sqrt{\dfrac{x^2 + y^2}{24}} \le z \le 0,$
$y \ge -\dfrac{x}{\sqrt{3}}, \quad y \ge -\sqrt{3}x.$

15.9. $4 \le x^2 + y^2 + z^2 \le 64,$
$-\sqrt{\dfrac{x^2 + y^2}{35}} \le z \le \sqrt{\dfrac{x^2 + y^2}{3}},$
$x \le y \le 0.$

15.10. $16 \le x^2 + y^2 + z^2 \le 100,$
$\sqrt{\dfrac{x^2 + y^2}{15}} \le z \le \sqrt{\dfrac{x^2 + y^2}{3}},$
$\sqrt{3}x \le y \le 0.$

15.11.
$$16 \le x^2 + y^2 + z^2 \le 100,$$
$$z \le \sqrt{\frac{x^2 + y^2}{3}}, \quad -\sqrt{3}x \le y \le -\frac{x}{\sqrt{3}}.$$

15.12.
$$16 \le x^2 + y^2 + z^2 \le 64,$$
$$z \ge -\sqrt{\frac{x^2 + y^2}{63}},$$
$$-\frac{x}{\sqrt{3}} \le y \le -\sqrt{3}x.$$

15.13.
$$4 \le x^2 + y^2 + z^2 \le 49,$$
$$z \ge \sqrt{\frac{x^2 + y^2}{99}}, \quad y \le 0, \quad y \le \sqrt{3}x.$$

15.14.
$$36 \le x^2 + y^2 + z^2 \le 121,$$
$$z \ge -\sqrt{\frac{x^2 + y^2}{99}}, \quad y \ge 0, \quad y \ge \sqrt{3}x.$$

15.15.
$$4 \le x^2 + y^2 + z^2 \le 64,$$
$$0 \le z \le \sqrt{\frac{x^2 + y^2}{24}},$$
$$y \le \sqrt{3}x, \quad y \le \frac{x}{\sqrt{3}}.$$

15.16.
$$36 \le x^2 + y^2 + z^2 \le 144,$$
$$-\sqrt{\frac{x^2 + y^2}{24}} \le z \le 0,$$
$$y \ge \sqrt{3}x, \quad y \ge \frac{x}{\sqrt{3}}.$$

15.17.
$$9 \le x^2 + y^2 + z^2 \le 81,$$
$$-\sqrt{\frac{x^2 + y^2}{3}} \le z \le \sqrt{\frac{x^2 + y^2}{35}},$$
$$0 \le y \le -x.$$

15.18.
$$36 \le x^2 + y^2 + z^2 \le 144,$$
$$-\sqrt{\frac{x^2 + y^2}{3}} \le z \le \sqrt{\frac{x^2 + y^2}{35}},$$
$$0 \le y \le -\sqrt{3}x.$$

15.19.
$$36 \le x^2 + y^2 + z^2 \le 144,$$
$$z \le \sqrt{\frac{x^2 + y^2}{3}}, \quad \sqrt{3}x \le y \le \frac{x}{\sqrt{3}}.$$

15.20.
$$36 \le x^2 + y^2 + z^2 \le 100,$$
$$z \ge -\sqrt{\frac{x^2 + y^2}{63}},$$
$$\frac{x}{\sqrt{3}} \le y \le \sqrt{3}x.$$

15.21.
$$9 \le x^2 + y^2 + z^2 \le 64,$$
$$z \ge \sqrt{\frac{x^2 + y^2}{99}},$$
$$y \le \frac{x}{\sqrt{3}}, \quad y \le -\frac{x}{\sqrt{3}}.$$

15.22.
$$49 \le x^2 + y^2 + z^2 \le 144,$$
$$z \le -\sqrt{\frac{x^2 + y^2}{99}},$$
$$y \ge \frac{x}{\sqrt{3}}, \quad y \ge -\frac{x}{\sqrt{3}}.$$

15.23. $9 \le x^2 + y^2 + z^2 \le 81$, $0 \le z \le \sqrt{\dfrac{x^2+y^2}{24}}$, $y \le 0$, $y \le \dfrac{x}{\sqrt{3}}$.

15.24. $49 \le x^2 + y^2 + z^2 \le 81$, $-\sqrt{\dfrac{x^2+y^2}{24}} \le z \le 0$, $y \ge 0$, $y \ge \dfrac{x}{\sqrt{3}}$.

15.25. $16 \le x^2 + y^2 + z^2 \le 100$, $-\sqrt{\dfrac{x^2+y^2}{3}} \le z \le \sqrt{\dfrac{x^2+y^2}{35}}$, $0 \le y \le x$.

15.26. $64 \le x^2 + y^2 + z^2 \le 196$, $-\sqrt{\dfrac{x^2+y^2}{3}} \le z \le -\sqrt{\dfrac{x^2+y^2}{15}}$, $0 \le y \le \sqrt{3}x$.

15.27. $64 \le x^2 + y^2 + z^2 \le 196$, $z \le \sqrt{\dfrac{x^2+y^2}{3}}$, $\dfrac{x}{\sqrt{3}} \le y \le 0$.

15.28. $64 \le x^2 + y^2 + z^2 \le 144$, $z \ge -\sqrt{\dfrac{x^2+y^2}{63}}$, $0 \le y \le \dfrac{x}{\sqrt{3}}$.

15.29. $16 \le x^2 + y^2 + z^2 \le 81$, $z \ge \sqrt{\dfrac{x^2+y^2}{99}}$, $y \le 0$, $y \le -\sqrt{3}x$.

15.30. $64 \le x^2 + y^2 + z^2 \le 169$, $z \le -\sqrt{\dfrac{x^2+y^2}{99}}$, $y \ge 0$, $y \ge -\sqrt{3}x$.

15.31. $16 \le x^2 + y^2 + z^2 \le 100$, $0 \le z \le \sqrt{\dfrac{x^2+y^2}{24}}$, $y \le 0$, $y \le \dfrac{x}{\sqrt{3}}$.

Задача 16. Тело V задано ограничивающими его поверхностями, μ - плотность.

Найти массу тела.

$$64\left(x^2 + y^2\right) = z^2, \quad x^2 + y^2 = 4,$$

16.1. $y = 0, \quad z = 0 \quad \left(y \geq 0, \ z \geq 0\right),$

$\mu = 5\left(x^2 + y^2\right)\big/4.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 1,$$

16.2. $\left(x^2 + y^2 \leq 1\right), \quad x = 0 \quad \left(x \geq 0\right);$

$\mu = 4|z|.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = 2z,$$

16.3. $x = 0, \ y = 0, \ z = 0 \quad \left(x \geq 0, \ y \geq 0\right);$

$\mu = 10x.$

$$x^2 + y^2 = \frac{16}{49}z^2, \quad x^2 + y^2 = \frac{4}{7}z,$$

16.4. $x = 0, \ y = 0, \quad \left(x \geq 0, \ y \geq 0\right);$

$\mu = 80\,yz.$

$$x^2 + y^2 + z^2 = 1, \quad x^2 + y^2 = 4z^2,$$

16.5. $x = 0, \ y = 0, \quad \left(x \geq 0, \ y \geq 0, \ z \geq 0\right);$

$\mu = 20z.$

$$36\left(x^2 + y^2\right) = z^2, \quad x^2 + y^2 = 1,$$

16.6. $x = 0, \quad z = 0 \quad \left(x \geq 0, \ z \geq 0\right),$

$\mu = \frac{5}{6}\left(x^2 + y^2\right).$

$$x^2 + y^2 + z^2 = 16, \quad x^2 + y^2 = 4,$$

16.7. $\left(x^2 + y^2 \leq 4\right);$

$\mu = 2|z|.$

$$x^2 + y^2 = 4, \quad x^2 + y^2 = 8z,$$

16.8. $x = 0, \ y = 0, \ z = 0 \ \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 5x.$

$$x^2 + y^2 = \frac{4}{25}z^2, \quad x^2 + y^2 = \frac{2}{5}z,$$

16.9. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 28xz.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = z^2,$$

16.10. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0, \ z \geq 0 \right);$

$\mu = 6z.$

$$25 \left(x^2 + y^2 \right) = z^2, \quad x^2 + y^2 = 4,$$

$$x = 0, \quad y = 0, \ z = 0$$

16.11. $\left(x \geq 0, \ y \geq 0, \ z \geq 0 \right),$

$\mu = 2 \left(x^2 + y^2 \right).$

$$x^2 + y^2 + z^2 = 9, \quad x^2 + y^2 = 4,$$

16.12. $\left(x^2 + y^2 \leq 4 \right), \quad y = 0 \ \left(y \geq 0 \right);$

$\mu = |z|.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = 6z,$$

16.13. $x = 0, \ y = 0, \ z = 0 \ \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 90y.$

$$x^2 + y^2 = \frac{1}{25}z^2, \quad x^2 + y^2 = \frac{1}{5}z,$$

16.14. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 14yz.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 9z^2,$$

16.15. $x = 0, \quad y = 0, \quad \left(x \geq 0, \quad y \geq 0, \quad z \geq 0\right);$

$\mu = 10z.$

$$9\left(x^2 + y^2\right) = z^2, \quad x^2 + y^2 = 4,$$

$x = 0, \quad y = 0, \quad z = 0$

16.16. $\left(x \geq 0, \quad y \geq 0, \quad z \geq 0\right),$

$\mu = 5\left(x^2 + y^2\right)/3.$

$$x^2 + y^2 + z^2 = 4,$$

16.17. $x^2 + y^2 = 1, \quad \left(x^2 + y^2 \leq 1\right);$

$\mu = |z|.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = z,$$

$x = 0, \quad y = 0, \quad z = 0,$

16.18. $\left(x \geq 0, \quad y \geq 0\right);$

$\mu = 10y.$

$$x^2 + y^2 = \frac{1}{49}z^2, \quad x^2 + y^2 = \frac{1}{7}z,$$

16.19. $x = 0, \quad y = 0, \quad \left(x \geq 0, \quad y \geq 0\right);$

$\mu = 10xz.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 4z^2,$$

16.20. $x = 0, \quad y = 0, \quad \left(x \geq 0, \quad y \geq 0, \quad z \geq 0\right);$

$\mu = 10z.$

$$16\left(x^2 + y^2\right) = z^2, \quad x^2 + y^2 = 1,$$

16.21. $x = 0, \quad y = 0, \quad z = 0 \quad \left(x \geq 0, \quad y \geq 0, \quad z \geq 0\right),$

$\mu = 5\left(x^2 + y^2\right).$

$$x^2 + y^2 + z^2 = 16,$$

16.22. $x^2 + y^2 = 4$ $\left(x^2 + y^2 \le 4\right);$

$\mu = |z|.$

$$x^2 + y^2 = 4, \quad x^2 + y^2 = 4z,$$

16.23. $x = 0, \ y = 0, \ z = 0 \ \left(x \ge 0, \ y \ge 0\right);$

$\mu = 5y.$

$$x^2 + y^2 = z^2, \quad x^2 + y^2 = z,$$

16.24. $x = 0, \ y = 0, \ \left(x \ge 0, \ y \ge 0\right);$

$\mu = 35yz.$

$$x^2 + y^2 + z^2 = 1, \quad x^2 + y^2 = z^2,$$

16.25. $x = 0, \ y = 0, \ \left(x \ge 0, \ y \ge 0, \ z \ge 0\right);$

$\mu = 32z.$

$$x^2 + y^2 = z^2, \quad x^2 + y^2 = 4,$$

$$x = 0, \quad y = 0, \ z = 0$$

16.26. $\left(x \ge 0, \ y \ge 0, \ z \ge 0\right),$

$\mu = 5\left(x^2 + y^2\right)/2.$

$$x^2 + y^2 + z^2 = 9, \quad x^2 + y^2 = 4,$$

16.27. $\left(x^2 + y^2 \le 4\right), \quad z = 0 \ \left(z \ge 0\right);$

$\mu = 2z.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = 3z,$$

$$x = 0, \ y = 0, \ z = 0$$

16.28. $\left(x \ge 0, \ y \ge 0\right);$

$\mu = 15x.$

$$x^2 + y^2 = \frac{4}{49}z^2, \quad x^2 + y^2 = \frac{2}{7}z,$$

16.29. $x = 0, \; y = 0, \; (x \geq 0, \; y \geq 0)$;

$\mu = 20xz.$

$$x^2 + y^2 + z^2 = 16, \quad x^2 + y^2 = 9z^2,$$

16.30. $x = 0, \; y = 0, \; (x \geq 0, \; y \geq 0, \; z \geq 0)$;

$\mu = 5z.$

$$4\left(x^2 + y^2\right) = z^2, \quad x^2 + y^2 = 1,$$

16.31. $y = 0, \; z = 0 \; (y \geq 0, \; z \geq 0),$

$\mu = 10\left(x^2 + y^2\right).$