IV. ИНТЕГРАЛЫ

Расчетные задания

Задача 1. Вычислить неопределенные интегралы.

1.1. $\int (4-3x)e^{-3x}dx.$

1.2. $\int \operatorname{arctg}\sqrt{4x-1}dx.$

1.3. $\int (3x+4)e^{3x}dx.$

1.4. $\int (4x-2)\cos 2xdx.$

1.5. $\int (4-16x)\sin 4xdx.$

1.6. $\int (5x-2)e^{3x}dx.$

1.7. $\int (1-6x)e^{2x}dx.$

1.8. $\int \ln(x^2+4)dx.$

1.9. $\int \ln(4x^2+1)dx.$

1.10. $\int (2-4x)\sin 2xdx.$

1.11. $\int \operatorname{arctg}\sqrt{6x-1}dx.$

1.12. $\int e^{-2x}(4x-3)dx.$

1.13. $\int e^{-3x}(2-9x)dx.$

1.14. $\int \operatorname{arctg}\sqrt{2x-1}dx.$

1.15. $\int \operatorname{arctg}\sqrt{3x-1}dx.$

1.16. $\int \operatorname{arctg}\sqrt{5x-1}dx.$

1.17. $\int (5x+6)\cos 2xdx.$

1.18. $\int (3x-2)\cos 5xdx.$

1.19. $\int (x\sqrt{2}-3)\cos 2xdx.$

1.20. $\int (4x+7)\cos 3xdx.$

1.21. $\int (2x-5)\cos 4xdx.$

1.22. $\int (8-3x)\cos 5xdx.$

1.23. $\int (x+5)\sin 3xdx.$

1.24. $\int (2-3x)\sin 2xdx.$

1.25. $\int (4x+3)\sin 5xdx.$

1.26. $\int (7x-10)\sin 4xdx.$

1.27. $\int (\sqrt{2}-8x)\sin 3xdx.$

1.28. $\int \dfrac{xdx}{\cos^2 x}.$

1.29. $\int \dfrac{xdx}{\sin^2 x}.$

1.30. $\int x\sin^2 xdx.$

1.31. $\int \dfrac{x\cos xdx}{\sin^3 x}.$

Задача 2. Вычислить определенные интегралы.

2.1. $\int\limits_{-2}^{0} (x^2 + 5x + 6)\cos 2x\,dx.$

2.2. $\int\limits_{-2}^{0} (x^2 - 4)\cos 3x\,dx.$

2.3. $\int\limits_{-1}^{0} (x^2 + 4x + 3)\cos x\,dx.$

2.4. $\int\limits_{-2}^{0} (x + 2)^2 \cos 3x\,dx.$

2.5. $\int\limits_{-4}^{0} (x^2 + 7x + 12)\cos x\,dx.$

2.6. $\int\limits_{0}^{\pi} (2x^2 + 4x + 7)\cos 2x\,dx.$

2.7. $\int\limits_{0}^{\pi} (9x^2 + 9x + 11)\cos 3x\,dx.$

2.8. $\int\limits_{0}^{\pi} (8x^2 + 16x + 17)\cos 4x\,dx.$

2.9. $\int\limits_{0}^{2\pi} (3x^2 + 5)\cos 2x\,dx.$

2.10. $\int\limits_{0}^{2\pi} (2x^2 - 15)\cos 3x\,dx.$

2.11. $\int\limits_{0}^{2\pi} (3 - 7x^2)\cos 2x\,dx.$

2.12. $\int\limits_{0}^{2\pi} (1 - 8x^2)\cos 4x\,dx.$

2.13. $\int\limits_{-1}^{0} (x^2 + 2x + 1)\sin 3x\,dx.$

2.14. $\int\limits_{0}^{3} (x^2 - 3x)\sin 2x\,dx.$

2.15. $\int\limits_{0}^{\pi} (x^2 - 3x + 2)\sin x\,dx.$

2.16. $\int\limits_{0}^{\frac{\pi}{2}} (x^2 - 5x + 6)\sin 3x\,dx.$

2.17. $\int\limits_{-3}^{0} (x^2 + 6x + 9)\sin 2x\,dx.$

2.18. $\int\limits_{0}^{\frac{\pi}{4}} (x^2 + 17,5)\sin 2x\,dx.$

2.19. $\int\limits_{0}^{\frac{\pi}{2}} (1 - 5x^2)\sin x\,dx.$

2.20. $\int\limits_{\frac{\pi}{4}}^{3} (3x - x^2)\sin 2x\,dx.$

2.21. $\int\limits_{1}^{2} x\ln^2 x\,dx.$

2.22. $\int\limits_{1}^{e^2} \dfrac{\ln^2 x\,dx}{\sqrt{x}}.$

2.23. $\int\limits_{1}^{8} \dfrac{\ln^2 x\,dx}{\sqrt[3]{x^2}}.$

2.24. $\int\limits_{0}^{1} (x + 1)\ln^2 (x + 1)\,dx.$

$2.25.\ \int\limits_{2}^{3}(x-1)^3 \ln^2(x-1)dx.$

$2.26.\ \int\limits_{-1}^{0}(x+2)^3 \ln^2(x+2)dx.$

$2.27.\ \int\limits_{0}^{2}(x+1)^2 \ln^2(x+1)dx.$

$2.28.\ \int\limits_{1}^{e}\sqrt{x}\ln^2 xdx.$

$2.29.\ \int\limits_{-1}^{1} x^2\, e^{-\frac{x}{2}}\, dx.$

$2.30.\ \int\limits_{0}^{1} x^2\, e^{3x}\, dx.$

$2.31.\ \int\limits_{-2}^{0}(x^2+2)e^{\frac{x}{2}}\, dx.$

Задача 3. Найти неопределенные интегралы.

$3.1.\ \int \dfrac{dx}{x\sqrt{x^2+1}}.$

$3.2.\ \int \dfrac{1+\ln x}{x}dx.$

$3.3.\ \int \dfrac{dx}{x\sqrt{x^2-1}}.$

$3.4.\ \int \dfrac{x^2+\ln x^2}{x}dx.$

$3.5.\ \int \dfrac{xdx}{\sqrt{x^4+x^2+1}}.$

$3.6.\ \int \dfrac{(\arccos x)^3-1}{\sqrt{1-x^2}}dx.$

$3.7.\ \int \operatorname{tg} x \ln \cos xdx.$

$3.8.\ \int \dfrac{\operatorname{tg}(x+1)}{\cos^2(x+1)}dx.$

$3.9.\ \int \dfrac{x^3}{(x^2+1)^2}dx.$

$3.10.\ \int \dfrac{1-\cos x}{(x-\sin x)^2}dx.$

$3.11.\ \int \dfrac{\sin x-\cos x}{(\cos x+\sin x)^5}dx.$

$3.12.\ \int \dfrac{x\cos x+\sin x}{(x\sin x)^2}dx.$

$3.13.\ \int \dfrac{x^3+x}{x^4+1}dx.$

$3.14.\ \int \dfrac{xdx}{\sqrt{x^4-x^2-1}}.$

$3.15.\ \int \dfrac{xdx}{\sqrt[3]{x-1}}.$

$3.16.\ \int \dfrac{1+\ln(x-1)}{x-1}dx.$

$3.17.\ \int \dfrac{(x^2+1)dx}{(x^3+3x+1)^5}.$

$3.18.\ \int \dfrac{4\operatorname{arctg} x-x}{1+x^2}dx.$

3.19. $\displaystyle\int \frac{x^3}{x^2+4}dx.$

3.20. $\displaystyle\int \frac{x+\cos x}{x^2+2\sin x}dx.$

3.21. $\displaystyle\int \frac{2\cos x+3\sin x}{(2\sin x-3\cos x)^3}dx.$

3.22. $\displaystyle\int \frac{8x-\operatorname{arctg}2x}{1+4x^2}dx.$

3.23. $\displaystyle\int \frac{1/\left(2\sqrt{x}\right)+1}{\left(\sqrt{x}+x\right)^2}dx.$

3.24. $\displaystyle\int \frac{x}{x^4+1}dx.$

3.25. $\displaystyle\int \frac{x+1/x}{\sqrt{x^2+1}}dx.$

3.26. $\displaystyle\int \frac{x-1/x}{\sqrt{x^2+1}}dx.$

3.27. $\displaystyle\int \frac{\operatorname{arctg}x+x}{1+x^2}dx.$

3.28. $\displaystyle\int \frac{x-\left(\operatorname{arctg}x\right)^4}{1+x^2}dx.$

3.29. $\displaystyle\int \frac{x^3}{x^2+1}dx.$

3.30. $\displaystyle\int \frac{\left(\arcsin x\right)^2+1}{\sqrt{1-x^2}}dx.$

3.31. $\displaystyle\int \frac{1-\sqrt{x}}{\sqrt{x}(x+1)}dx.$

Задача 4. Вычислить определенные интегралы.

4.1. $\displaystyle\int_{e+1}^{e^2+1} \frac{1+\ln(x-1)}{x-1}dx.$

4.2. $\displaystyle\int_{0}^{1} \frac{\left(x^2+1\right)dx}{\left(x^3+3x+1\right)^2}.$

4.3. $\displaystyle\int_{0}^{1} \frac{4\operatorname{arctg}x-x}{1+x^2}dx.$

4.4. $\displaystyle\int_{0}^{2} \frac{x^3 dx}{x^2+4}.$

4.5. $\displaystyle\int_{\pi}^{2\pi} \frac{x+\cos x}{x^2+2\sin x}dx.$

4.6. $\displaystyle\int_{0}^{\pi/4} \frac{2\cos x+3\sin x}{(2\sin x-3\cos x)^3}dx.$

4.7. $\displaystyle\int_{0}^{1/2} \frac{8x-\operatorname{arctg}2x}{1+4x^2}dx.$

4.8. $\displaystyle\int_{1}^{4} \frac{1/\left(2\sqrt{x}\right)+1}{\left(\sqrt{x}+x\right)^2}dx.$

4.9. $\displaystyle\int_{0}^{1} \frac{xdx}{x^4+1}.$

4.10. $\displaystyle\int_{\sqrt{3}}^{\sqrt{8}} \frac{x+1/x}{\sqrt{x^2+1}}dx.$

4.11. $\displaystyle\int_{\sqrt{3}}^{\sqrt{8}} \frac{x-1/x}{\sqrt{x^2+1}}dx.$

4.12. $\displaystyle\int_{0}^{\sqrt{3}} \frac{\operatorname{arctg} x + x}{1+x^2}dx.$

4.13. $\displaystyle\int_{0}^{\sqrt{3}} \frac{x-(\operatorname{arctg} x)^4}{1+x^2}dx.$

4.14. $\displaystyle\int_{0}^{1} \frac{x^3}{x^2+1}dx.$

4.15. $\displaystyle\int_{0}^{\sin 1} \frac{(\arcsin x)^2+1}{\sqrt{1-x^2}}dx.$

4.16. $\displaystyle\int_{1}^{3} \frac{1-\sqrt{x}}{\sqrt{x}(x+1)}dx.$

4.17. $\displaystyle\int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{x\sqrt{x^2+1}}.$

4.18. $\displaystyle\int_{1}^{e} \frac{1+\ln x}{x}dx.$

4.19. $\displaystyle\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}.$

4.20. $\displaystyle\int_{1}^{e} \frac{x^2+\ln x^2}{x}dx.$

4.21. $\displaystyle\int_{0}^{1} \frac{xdx}{\sqrt{x^4+x^2+1}}.$

4.22. $\displaystyle\int_{0}^{1} \frac{x^3 dx}{(x^2+1)^2}.$

4.23. $\displaystyle\int_{0}^{\pi/4} \operatorname{tg} x \ln \cos x\, dx.$

4.24. $\displaystyle\int_{-1}^{0} \frac{\operatorname{tg}(x+1)}{\cos^2(x+1)}dx.$

4.25. $\displaystyle\int_{0}^{1/\sqrt{2}} \frac{(\arccos x)^3-1}{\sqrt{1-x^2}}dx.$

4.26. $\displaystyle\int_{\pi}^{2\pi} \frac{1-\cos x}{(x-\sin x)^2}dx.$

4.27. $\displaystyle\int_{0}^{\pi/4} \frac{\sin x - \cos x}{(\cos x + \sin x)^5}dx.$

4.28. $\displaystyle\int_{\pi/4}^{\pi/2} \frac{x\cos x + \sin x}{(x\sin x)^2}dx.$

4.29. $\displaystyle\int_{0}^{1} \frac{x^3+x}{x^4+1}dx.$

4.30. $\displaystyle\int_{\sqrt{2}}^{\sqrt{3}} \frac{xdx}{\sqrt{x^4-x^2-1}}.$

4.31. $\displaystyle\int_{2}^{9} \frac{xdx}{\sqrt[3]{x-1}}.$

Задача 5. Найти неопределенные интегралы.

5.1. $\displaystyle\int \frac{x^3+1}{x^2-x}dx.$

5.2. $\displaystyle\int \frac{3x^3+1}{x^2-1}dx.$

5.3. $\displaystyle\int \frac{x^3-17}{x^2-4x+3}dx.$

5.4. $\displaystyle\int \frac{2x^3+5}{x^2-x-2}dx.$

5.5. $\int \dfrac{2x^3-1}{x^2+x-6}dx.$

5.6. $\int \dfrac{3x^3+25}{x^2+3x+2}dx.$

5.7. $\int \dfrac{x^3+2x^2+3}{(x-1)(x-2)(x-3)}dx.$

5.8. $\int \dfrac{3x^3+2x^2+1}{(x+2)(x-2)(x-1)}dx.$

5.9. $\int \dfrac{x^3}{(x-1)(x+1)(x+2)}dx.$

5.10. $\int \dfrac{x^3-3x^2-12}{(x-4)(x-3)(x-2)}dx.$

5.11. $\int \dfrac{x^3-3x^2-12}{(x-4)(x-3)x}dx.$

5.12. $\int \dfrac{4x^3+x^2+2}{x(x-1)(x-2)}dx.$

5.13. $\int \dfrac{3x^3-2}{x^3-x}dx.$

5.14. $\int \dfrac{x^3-3x^2-12}{(x-4)(x-2)x}dx.$

5.15. $\int \dfrac{x^5-x^3+1}{x^2-x}dx.$

5.16. $\int \dfrac{x^5+3x^3-1}{x^2+x}dx.$

5.17. $\int \dfrac{2x^5-8x^3+3}{x^2-2x}dx.$

5.18. $\int \dfrac{3x^5-12x^3-7}{x^2+2x}dx.$

5.19. $\int \dfrac{-x^5+9x^3+4}{x^2+3x}dx.$

5.20. $\int \dfrac{-x^5+25x^3+1}{x^2+5x}dx.$

5.21. $\int \dfrac{x^3-5x^2+5x+23}{(x-1)(x+1)(x-5)}dx.$

5.22. $\int \dfrac{x^5+2x^4-2x^3+5x^2-7x+9}{(x+3)(x-1)x}dx.$

5.23. $\int \dfrac{2x^4-5x^2-8x-8}{x(x-2)(x+2)}dx.$

5.24. $\int \dfrac{4x^4+2x^2-x-3}{x(x-1)(x+1)}dx.$

5.25. $\int \dfrac{3x^4+3x^3-5x^2+2}{x(x-1)(x+2)}dx.$

5.26. $\int \dfrac{2x^4+2x^3-41x^2+20}{x(x-4)(x+5)}dx.$

5.27. $\int \dfrac{x^5-x^4-6x^3+13x+6}{x(x-3)(x+2)}dx.$

5.28. $\int \dfrac{3x^3-x^2-12x-2}{x(x+1)(x-2)}dx.$

5.29. $\int \dfrac{2x^4+2x^3-3x^2+2x-9}{x(x-1)(x+3)}dx.$

5.30. $\int \dfrac{2x^3-x^2-7x-12}{x(x-3)(x+1)}dx.$

5.31. $\int \dfrac{2x^3-40x-8}{x(x+4)(x-2)}dx.$

Задача 6. Найти неопределенные интегралы.

6.1. $\int \dfrac{x^3+6x^2+13x+9}{(x+1)(x+2)^3}dx.$

6.2. $\int \dfrac{x^3+6x^2+13x+8}{x(x+2)^3}dx.$

6.3. $\int \dfrac{x^3-6x^2+13x-6}{(x+2)(x-2)^3}dx.$

6.4. $\int \dfrac{x^3+6x^2+14x+10}{(x+1)(x+2)^3}dx.$

6.5. $\int \dfrac{x^3 - 6x^2 + 11x - 10}{(x+2)(x-2)^3} dx.$

6.6. $\int \dfrac{x^3 + 6x^2 + 11x + 7}{(x+1)(x+2)^3} dx.$

6.7. $\int \dfrac{2x^3 + 6x^2 + 7x + 1}{(x-1)(x+1)^3} dx.$

6.8. $\int \dfrac{x^3 + 6x^2 + 10x + 10}{(x-1)(x+2)^3} dx.$

6.9. $\int \dfrac{2x^3 + 6x^2 + 7x + 2}{x(x+1)^3} dx.$

6.10. $\int \dfrac{x^3 - 6x^2 + 13x - 8}{x(x-2)^3} dx.$

6.11. $\int \dfrac{x^3 - 6x^2 + 13x - 7}{(x+1)(x-2)^3} dx.$

6.12. $\int \dfrac{x^3 - 6x^2 + 14x - 6}{(x+1)(x-2)^3} dx.$

6.13. $\int \dfrac{x^3 - 6x^2 + 10x - 10}{(x+1)(x-2)^3} dx.$

6.14. $\int \dfrac{x^3 + x + 2}{(x+2)x^3} dx.$

6.15. $\int \dfrac{3x^3 + 9x^2 + 10x + 2}{(x-1)(x+1)^3} dx.$

6.16. $\int \dfrac{2x^3 + x + 1}{(x+1)x^3} dx.$

6.17. $\int \dfrac{2x^3 + 6x^2 + 7x + 4}{(x+2)(x+1)^3} dx.$

6.18. $\int \dfrac{2x^3 + 6x^2 + 5x}{(x+2)(x+1)^3} dx.$

6.19. $\int \dfrac{2x^3 + 6x^2 + 7x}{(x-2)(x+1)^3} dx.$

6.20. $\int \dfrac{2x^3 + 6x^2 + 5x + 4}{(x-2)(x+1)^3} dx.$

6.21. $\int \dfrac{x^3 + 6x^2 + 4x + 24}{(x-2)(x+2)^3} dx.$

6.22. $\int \dfrac{x^3 + 6x^2 + 14x + 4}{(x-2)(x+2)^3} dx.$

6.23. $\int \dfrac{x^3 + 6x^2 + 18x - 4}{(x-2)(x+2)^3} dx.$

6.24. $\int \dfrac{x^3 + 6x^2 + 10x + 12}{(x-2)(x+2)^3} dx.$

6.25. $\int \dfrac{x^3 - 6x^2 + 14x - 4}{(x+2)(x-2)^3} dx.$

6.26. $\int \dfrac{x^3 + 6x^2 + 15x + 2}{(x-2)(x+2)^3} dx.$

6.27. $\int \dfrac{2x^3 - 6x^2 + 7x - 4}{(x-2)(x-1)^3} dx.$

6.28. $\int \dfrac{2x^3 - 6x^2 + 7x}{(x+2)(x-1)^3} dx.$

6.29. $\int \dfrac{x^3 + 6x^2 - 10x + 52}{(x-2)(x+2)^3} dx.$

6.30. $\int \dfrac{x^3 - 6x^2 + 13x - 6}{(x+2)(x-2)^3} dx.$

6.31. $\int \dfrac{x^3 + 6x^2 + 13x + 6}{(x-2)(x+2)^3} dx.$

Задача 7. Найти неопределенные интегралы.

7.1. $\displaystyle\int \frac{x^3+4x^2+4x+2}{(x+1)^2(x^2+x+1)}dx.$

7.2. $\displaystyle\int \frac{x^3+4x^2+3x+2}{(x+1)^2(x^2+1)}dx.$

7.3. $\displaystyle\int \frac{2x^3+7x^2+7x-1}{(x+2)^2(x^2+x+1)}dx.$

7.4. $\displaystyle\int \frac{2x^3+4x^2+2x-1}{(x+1)^2(x^2+2x+2)}dx.$

7.5. $\displaystyle\int \frac{x^3+6x^2+9x+6}{(x+1)^2(x^2+2x+2)}dx.$

7.6. $\displaystyle\int \frac{2x^3+11x^2+16x+10}{(x+2)^2(x^2+2x+3)}dx.$

7.7. $\displaystyle\int \frac{3x^3+6x^2+5x-1}{(x+1)^2(x^2+2)}dx.$

7.8. $\displaystyle\int \frac{x^3+9x^2+21x+21}{(x+3)^2(x^2+3)}dx.$

7.9. $\displaystyle\int \frac{x^3+6x^2+8x+8}{(x+2)^2(x^2+4)}dx.$

7.10. $\displaystyle\int \frac{x^3+5x^2+12x+4}{(x+2)^2(x^2+4)}dx.$

7.11. $\displaystyle\int \frac{2x^3-4x^2-16x-12}{(x-1)^2(x^2+4x+5)}dx.$

7.12. $\displaystyle\int \frac{-3x^3+13x^2-13x+1}{(x-2)^2(x^2-x+1)}dx.$

7.13. $\displaystyle\int \frac{x^3+2x^2+10x}{(x+1)^2(x^2-x+1)}dx.$

7.14. $\displaystyle\int \frac{3x^3+x+46}{(x-1)^2(x^2+9)}dx.$

7.15. $\displaystyle\int \frac{4x^3+24x^2+20x-28}{(x+3)^2(x^2+2x+2)}dx.$

7.16. $\displaystyle\int \frac{2x^3+3x^2+3x+2}{(x^2+x+1)(x^2+1)}dx.$

7.17. $\displaystyle\int \frac{x^3+x+1}{(x^2+x+1)(x^2+1)}dx.$

7.18. $\displaystyle\int \frac{x^2+x+3}{(x^2+x+1)(x^2+1)}dx.$

7.19. $\displaystyle\int \frac{2x^3+4x^2+2x+2}{(x^2+x+1)(x^2+x+2)}dx.$

7.20. $\displaystyle\int \frac{2x^3+7x^2+7x+9}{(x^2+x+1)(x^2+x+2)}dx.$

7.21. $\displaystyle\int \frac{4x^2+3x+4}{(x^2+1)(x^2+x+1)}dx.$

7.22. $\displaystyle\int \frac{3x^3+4x^2+6x}{(x^2+2)(x^2+2x+2)}dx.$

7.23. $\displaystyle\int \frac{2x^2-x+1}{(x^2-x+1)(x^2+1)}dx.$

7.24. $\displaystyle\int \frac{x^3+x^2+1}{(x^2-x+1)(x^2+1)}dx.$

7.25. $\displaystyle\int \frac{x^3 + x + 1}{\left(x^2 - x + 1\right)\left(x^2 + 1\right)}dx.$

7.26. $\displaystyle\int \frac{2x^3 + 2x + 1}{\left(x^2 - x + 1\right)\left(x^2 + 1\right)}dx.$

7.28. $\displaystyle\int \frac{x^3 + 2x^2 + x + 1}{\left(x^2 + x + 1\right)\left(x^2 + 1\right)}dx.$

7.29. $\displaystyle\int \frac{x + 4}{\left(x^2 + x + 2\right)\left(x^2 + 2\right)}dx.$

7.30. $\displaystyle\int \frac{2x^3 + 2x^2 + 2x + 1}{\left(x^2 + x + 1\right)\left(x^2 + 1\right)}dx.$

7.30. $\displaystyle\int \frac{3x^3 + 7x^2 + 12x + 6}{\left(x^2 + x + 3\right)\left(x^2 + 2x + 3\right)}dx.$

7.31. $\displaystyle\int \frac{2x^3 + 3x^2 + 3x + 2}{\left(x^2 + x + 1\right)\left(x^2 + 1\right)}dx.$

Задача 8. Вычислить определенные интегралы.

8.1. $\displaystyle\int_{\pi/2}^{2\,\mathrm{arctg}\,2} \frac{dx}{\sin^2 x\left(1 - \cos x\right)}.$

8.2. $\displaystyle\int_{0}^{\pi/2} \frac{\cos x\,dx}{2 + \cos x}.$

8.3. $\displaystyle\int_{\pi/2}^{2\,\mathrm{arctg}\,2} \frac{dx}{\sin^2 x\left(1 + \cos x\right)}.$

8.4. $\displaystyle\int_{2\,\mathrm{arctg}(1/2)}^{\pi/2} \frac{\cos x\,dx}{\left(1 - \cos x\right)^3}.$

8.5. $\displaystyle\int_{0}^{\pi/2} \frac{\cos x - \sin x}{\left(1 + \sin x\right)^2}dx.$

8.6. $\displaystyle\int_{2\,\mathrm{arctg}\,2}^{2\,\mathrm{arctg}\,3} \frac{dx}{\cos x\left(1 - \cos x\right)}.$

8.7. $\displaystyle\int_{2\,\mathrm{arctg}(1/3)}^{2\,\mathrm{arctg}(1/2)} \frac{dx}{\sin x\left(1 - \sin x\right)}.$

8.8. $\displaystyle\int_{2\,\mathrm{arctg}(1/2)}^{\pi/2} \frac{dx}{\left(1 + \sin x - \cos x\right)^2}.$

8.9. $\displaystyle\int_{0}^{\pi/2} \frac{\cos x\,dx}{5 + 4\cos x}.$

8.10. $\displaystyle\int_{0}^{2\pi/3} \frac{1 + \sin x}{1 + \cos x + \sin x}dx.$

8.11. $\displaystyle\int_{\pi/3}^{\pi/2} \frac{\cos x\,dx}{1 + \sin x - \cos x}.$

8.12. $\displaystyle\int_{0}^{\pi/2} \frac{\left(1 + \cos x\right)dx}{1 + \sin x + \cos x}.$

8.13. $\displaystyle\int_{0}^{\pi/2} \frac{\sin\,dx}{1 + \sin x + \cos x}.$

8.14. $\displaystyle\int_{0}^{2\,\mathrm{arctg}(1/2)} \frac{1 + \sin x}{\left(1 - \sin x\right)^2}dx.$

8.15. $\displaystyle\int_{0}^{\pi/2} \frac{\cos x\,dx}{1 + \sin x + \cos x}.$

8.16. $\displaystyle\int_{0}^{2\,\mathrm{arctg}(1/3)} \frac{\cos x\,dx}{\left(1 - \sin x\right)\left(1 + \cos x\right)}.$

8.17. $\displaystyle\int_{-2\pi/3}^{0} \frac{\cos x\, dx}{1+\cos x - \sin x}.$

8.18. $\displaystyle\int_{-\pi/2}^{0} \frac{\cos x\, dx}{\left(1+\cos x - \sin x\right)^2}.$

8.19. $\displaystyle\int_{0}^{\pi/2} \frac{\cos x\, dx}{\left(1+\cos x + \sin x\right)^2}.$

8.20. $\displaystyle\int_{0}^{2\,\mathrm{arctg}(1/2)} \frac{\left(1-\sin x\right) dx}{\cos x\left(1+\cos x\right)}.$

8.21. $\displaystyle\int_{0}^{\pi/2} \frac{\sin x\, dx}{\left(1+\sin x\right)^2}.$

8.22. $\displaystyle\int_{0}^{\pi/2} \frac{\sin x\, dx}{\left(1+\cos x + \sin x\right)^2}.$

8.23. $\displaystyle\int_{-\pi/2}^{0} \frac{\sin x\, dx}{\left(1+\cos x - \sin x\right)^2}.$

8.24. $\displaystyle\int_{-2\pi/3}^{0} \frac{\cos^2 x\, dx}{\left(1+\cos x - \sin x\right)^2}.$

8.25. $\displaystyle\int_{0}^{\pi/2} \frac{\sin^2 x\, dx}{\left(1+\cos x + \sin x\right)^2}.$

8.26. $\displaystyle\int_{0}^{2\pi/3} \frac{\cos^2 x\, dx}{\left(1+\cos x + \sin x\right)^2}.$

8.27. $\displaystyle\int_{\pi/2}^{2\,\mathrm{arctg}\,2} \frac{dx}{\sin x\left(1+\sin x\right)}.$

8.28. $\displaystyle\int_{0}^{\pi/2} \frac{dx}{\left(1+\cos x + \sin x\right)^2}.$

8.29. $\displaystyle\int_{0}^{\pi/2} \frac{\sin x\, dx}{2+\sin x}.$

8.30. $\displaystyle\int_{0}^{\pi/4} \frac{dx}{\cos x\left(1+\cos x\right)}.$

8.31. $\displaystyle\int_{0}^{\pi/2} \frac{\sin x\, dx}{5+3\sin x}.$

Задача 9. Вычислить определенные интегралы.

9.1. $\displaystyle\int_{\pi/4}^{\mathrm{arctg}\,3} \frac{dx}{\left(3\,\mathrm{tg}\,x + 5\right)\sin 2x}.$

9.2. $\displaystyle\int_{\mathrm{arccos}\left(4/\sqrt{17}\right)}^{\pi/4} \frac{2\,\mathrm{ctg}\,x + 1}{\left(2\sin x + \cos x\right)^2}\,dx.$

9.3. $\displaystyle\int_{0}^{\mathrm{arccos}\left(1/\sqrt{17}\right)} \frac{3+2\,\mathrm{tg}\,x}{2\sin^2 x + 3\cos^2 x - 1}\,dx.$

9.4. $\displaystyle\int_{\pi/4}^{\mathrm{arctg}\,3} \frac{4\,\mathrm{tg}\,x - 5}{1 - \sin 2x + 4\cos^2 x}\,dx.$

9.5. $\displaystyle\int_{0}^{\mathrm{arctg}(1/3)} \frac{\left(8 + \mathrm{tg}\,x\right)}{18\sin^2 x + 2\cos^2 x}\,dx.$

9.6. $\displaystyle\int_{0}^{\mathrm{arccos}\sqrt{2/3}} \frac{\mathrm{tg}\,x + 2}{\sin^2 x + 2\cos^2 x - 3}\,dx.$

9.7. $\displaystyle\int_{\mathrm{arcsin}\left(1/\sqrt{37}\right)}^{\pi/4} \frac{6\,\mathrm{tg}\,x\, dx}{3\sin 2x + 5\cos^2 x}.$

9.8. $\displaystyle\int_{0}^{\pi/4} \frac{2\,\mathrm{tg}^2 x - 11\,\mathrm{tg}\,x - 22}{4 - \mathrm{tg}\,x}\,dx.$

9.9. $\displaystyle\int_{-\operatorname{arctg}(1/3)}^{0} \frac{3\operatorname{tg} x + 1}{2\sin 2x - 5\cos 2x + 1}\,dx.$

9.10. $\displaystyle\int_{\pi/4}^{\operatorname{arctg} 3} \frac{1 + \operatorname{ctg} x}{\left(\sin x + 2\cos x\right)^2}\,dx.$

9.11. $\displaystyle\int_{\pi/4}^{\arccos\left(1/\sqrt{3}\right)} \frac{\operatorname{tg} x}{\sin^2 x - 5\cos^2 x + 4}\,dx.$

9.12. $\displaystyle\int_{0}^{\pi/4} \frac{6\sin^2 x}{3\cos 2x - 4}\,dx.$

9.13. $\displaystyle\int_{0}^{\operatorname{arctg} 3} \frac{4 + \operatorname{tg} x}{2\sin^2 x + 18\cos^2 x}\,dx.$

9.14. $\displaystyle\int_{0}^{\operatorname{arctg} 2} \frac{12 + \operatorname{tg} x}{3\sin^2 x + 12\cos^2 x}\,dx.$

9.15. $\displaystyle\int_{0}^{\operatorname{arctg}(2/3)} \frac{6 + \operatorname{tg} x}{9\sin^2 x + 4\cos^2 x}\,dx.$

9.16. $\displaystyle\int_{0}^{\arcsin\sqrt{3/7}} \frac{\operatorname{tg}^2 x\,dx}{3\sin^2 x + 4\cos^2 x - 7}.$

9.17. $\displaystyle\int_{0}^{\pi/4} \frac{7 + 3\operatorname{tg} x}{\left(\sin x + 2\cos x\right)^2}\,dx.$

9.18. $\displaystyle\int_{\arcsin\left(2/\sqrt{5}\right)}^{\arcsin\left(3/\sqrt{10}\right)} \frac{2\operatorname{tg} x + 5}{\left(5 - \operatorname{tg} x\right)\sin 2x}\,dx.$

9.19. $\displaystyle\int_{-\arccos\left(1/\sqrt{10}\right)}^{0} \frac{3\operatorname{tg}^2 x - 50}{2\operatorname{tg} x + 7}\,dx.$

9.20. $\displaystyle\int_{0}^{\pi/4} \frac{5\operatorname{tg} x + 2}{2\sin 2x + 5}\,dx.$

9.21. $\displaystyle\int_{\pi/4}^{\arcsin\left(2/\sqrt{5}\right)} \frac{4\operatorname{tg} x - 5}{4\cos^2 x - \sin 2x + 1}\,dx.$

9.22. $\displaystyle\int_{0}^{\arcsin\sqrt{7/8}} \frac{6\sin^2 x}{4 + 3\cos 2x}\,dx.$

9.23. $\displaystyle\int_{-\arccos\left(1/\sqrt{5}\right)}^{0} \frac{11 - 3\operatorname{tg} x}{\operatorname{tg} x + 3}\,dx.$

9.24. $\displaystyle\int_{0}^{\arcsin 3\sqrt{10}} \frac{2\operatorname{tg} x - 5}{\left(4\cos x - \sin x\right)^2}\,dx.$

9.25. $\displaystyle\int_{\pi/4}^{\arccos\left(1/\sqrt{26}\right)} \frac{dx}{\left(6 - \operatorname{tg} x\right)\sin 2x}.$

9.26. $\displaystyle\int_{0}^{\pi/4} \frac{4 - 7\operatorname{tg} x}{2 + 3\operatorname{tg} x}\,dx.$

9.27. $\displaystyle\int_{-\arcsin\left(2/\sqrt{5}\right)}^{\pi/4} \frac{2 - \operatorname{tg} x}{\left(\sin x + 3\cos x\right)^2}\,dx.$

9.28. $\displaystyle\int_{\pi/4}^{\arcsin\sqrt{2/3}} \frac{8\operatorname{tg} x\,dx}{3\cos^2 x + 8\sin 2x - 7}.$

9.29. $\displaystyle\int_{\arccos\left(1/\sqrt{10}\right)}^{\arccos\left(1/\sqrt{26}\right)} \frac{12\,dx}{\left(6 + 5\operatorname{tg} x\right)\sin 2x}.$

9.30. $\displaystyle\int_{0}^{\pi/3} \frac{\operatorname{tg}^2 x}{4 + 3\cos 2x}\,dx.$

9.31. $\displaystyle\int_{0}^{\arccos\left(1/\sqrt{6}\right)} \frac{3\operatorname{tg}^2 x - 1}{\operatorname{tg}^2 x + 5}.$

Задача 10. Вычислить определенные интегралы.

10.1. $\displaystyle\int_{\pi/2}^{\pi} 2^8 \sin^8 x \, dx.$

10.2. $\displaystyle\int_{0}^{\pi} 2^4 \sin^6 x \cos^2 x \, dx.$

10.3. $\displaystyle\int_{0}^{2\pi} \sin^4 x \cos^4 x \, dx.$

10.4. $\displaystyle\int_{0}^{2\pi} \sin^2 (x/4) \cos^6 (x/4) \, dx.$

10.5. $\displaystyle\int_{0}^{\pi} 2^4 \cos^8 (x/2) \, dx.$

10.6. $\displaystyle\int_{-\pi/2}^{0} 2^8 \sin^8 x \, dx.$

10.7. $\displaystyle\int_{\pi/2}^{\pi} 2^4 \sin^6 x \cos^2 x \, dx.$

10.8. $\displaystyle\int_{0}^{\pi} 2^4 \sin^4 x \cos^4 x \, dx.$

10.9. $\displaystyle\int_{0}^{2\pi} \sin^2 x \cos^6 x \, dx.$

10.10. $\displaystyle\int_{0}^{2\pi} \cos^8 (x/4) \, dx.$

10.11. $\displaystyle\int_{0}^{\pi} 2^4 \sin^8 (x/2) \, dx.$

10.12. $\displaystyle\int_{-\pi}^{0} 2^8 \sin^6 x \cos^2 x \, dx.$

10.13. $\displaystyle\int_{\pi/2}^{2\pi} 2^8 \sin^4 x \cos^4 x \, dx.$

10.14. $\displaystyle\int_{0}^{\pi} 2^4 \sin^2 x \cos^6 x \, dx.$

10.15. $\displaystyle\int_{0}^{2\pi} \cos^8 x \, dx.$

10.16. $\displaystyle\int_{0}^{2\pi} \sin^8 (x/4) \, dx.$

10.17. $\displaystyle\int_{0}^{\pi} 2^4 \sin^6 (x/2) \cos^2 (x/2) \, dx.$

10.18. $\displaystyle\int_{-\pi/2}^{0} 2^8 \sin^4 x \cos^4 x \, dx.$

10.19. $\displaystyle\int_{\pi/2}^{\pi} 2^8 \sin^2 x \cos^6 x \, dx.$

10.20. $\displaystyle\int_{0}^{\pi} 2^4 \cos^8 x \, dx.$

10.21. $\displaystyle\int_{0}^{2\pi} \sin^8 x \, dx.$

10.22. $\displaystyle\int_{0}^{2\pi} \sin^6 (x/4) \cos^2 (x/4) \, dx.$

10.23. $\displaystyle\int_{0}^{\pi} 2^4 \sin^4 (x/2) \cos^4 (x/2) \, dx.$

10.24. $\displaystyle\int_{-\pi/2}^{0} 2^8 \sin^2 x \cos^6 x \, dx.$

10.25. $\displaystyle\int_{\pi/2}^{2\pi} 2^8 \cos^8 x \, dx.$

10.26. $\displaystyle\int_{0}^{\pi} 2^4 \sin^8 x \, dx.$

10.27. $\displaystyle\int_0^{2\pi} \sin^6 x \cos^2 x \, dx.$

10.28. $\displaystyle\int_0^{2\pi} \sin^4(x/4)\cos^4(x/4)\, dx.$

10.29. $\displaystyle\int_0^{\pi} 2^4 \sin^2(x/2)\cos^6(x/2)\, dx.$

10.30. $\displaystyle\int_{-\pi/2}^{0} 2^8 \cos^8 x \, dx.$

10.31. $\displaystyle\int_0^{2\pi} \sin^4 3x \cos^4 3x \, dx.$

Задача 11. Вычислить определенные интегралы.

11.1. $\displaystyle\int_0^1 \frac{4\sqrt{1-x}-\sqrt{3x+1}}{\left(\sqrt{3x+1}+4\sqrt{1-x}\right)(3x+1)^2}\,dx.$

11.2. $\displaystyle\int_1^{64} \frac{1-\sqrt[6]{x}+2\sqrt[3]{x}}{x+2\sqrt{x^3}+\sqrt[3]{x^4}}\,dx.$

11.3. $\displaystyle\int_{-14/15}^{-7/8} \frac{6\sqrt{x+2}}{(x+2)^2\sqrt{x+1}}\,dx.$

11.4. $\displaystyle\int_6^9 \sqrt{\frac{9-2x}{2x-21}}\,dx.$

11.5. $\displaystyle\int_0^5 e^{\sqrt{\frac{5-x}{5+x}}}\,\frac{dx}{(5+x)\sqrt{25-x^2}}.$

11.6. $\displaystyle\int_8^{12} \sqrt{\frac{6-x}{x-14}}\,dx.$

11.7. $\displaystyle\int_0^1 e^{\sqrt{\frac{1-x}{1+x}}}\,\frac{dx}{(1+x)\sqrt{1-x^2}}.$

11.8. $\displaystyle\int_{5/2}^{10/3} \frac{\sqrt{x+2}+\sqrt{x-2}}{\left(\sqrt{x+2}-\sqrt{x-2}\right)(x-2)^2}\,dx.$

11.9. $\displaystyle\int_1^8 \frac{5\sqrt{x+24}}{(x+24)^2\sqrt{x}}\,dx.$

11.10. $\displaystyle\int_1^2 \frac{x+\sqrt{3x-2}-10}{\sqrt{3x-2}+7}\,dx.$

11.11. $\displaystyle\int_6^{10} \sqrt{\frac{4-x}{x-12}}\,dx.$

11.12. $\displaystyle\int_0^2 \frac{\left(4\sqrt{2-x}-\sqrt{2x+2}\right)dx}{\left(\sqrt{2x+2}+4\sqrt{2-x}\right)(2x+2)^2}.$

11.13. $\displaystyle\int_{-1/2}^{0} \frac{x\,dx}{2+\sqrt{2x+1}}.$

11.14. $\displaystyle\int_0^4 e^{\sqrt{\frac{4-x}{4+x}}}\,\frac{dx}{(4+x)\sqrt{16-x^2}}.$

11.15. $\displaystyle\int_{1/8}^{1} \frac{15\sqrt{x+3}}{(x+3)^2\sqrt{x}}\,dx.$

11.16. $\displaystyle\int_{-5/3}^{1} \frac{\sqrt[3]{3x+5}+2}{1+\sqrt[3]{3x+5}}\,dx.$

11.17. $\displaystyle\int_2^3 \sqrt{\frac{3-2x}{2x-7}}\,dx.$

11.18. $\displaystyle\int_0^7 \frac{\sqrt{x+25}}{(x+25)^2\sqrt{x+1}}\,dx.$

11.19. $\displaystyle\int_0^2 \frac{\left(4\sqrt{2-x}-\sqrt{3x+2}\right)dx}{\left(\sqrt{3x+2}+4\sqrt{2-x}\right)(3x+2)^2}.$

11.20. $\displaystyle\int_0^2 e^{\sqrt{\frac{2-x}{2+x}}} \frac{dx}{(2+x)\sqrt{4-x^2}}.$

11.21. $\displaystyle\int_3^5 \sqrt{\frac{2-x}{x-6}}\,dx.$

11.22. $\displaystyle\int_{1/24}^{1/3} \frac{5\sqrt{x+1}}{(x+1)^2\sqrt{x}}\,dx.$

11.23. $\displaystyle\int_9^{15} \sqrt{\frac{6-x}{x-18}}\,dx.$

11.24. $\displaystyle\int_0^1 \frac{\left(4\sqrt{1-x}-\sqrt{2x+1}\right)dx}{\left(\sqrt{2x+1}+4\sqrt{1-x}\right)(2x+1)^2}.$

11.25. $\displaystyle\int_1^{64} \frac{\left(2+\sqrt[3]{x}\right)dx}{\left(\sqrt[6]{x}+2\sqrt{x^3}+\sqrt{x}\right)\sqrt{x}}.$

11.26. $\displaystyle\int_{16/15}^{4/3} \frac{4\sqrt{x}}{x^2\sqrt{x-1}}\,dx.$

11.27. $\displaystyle\int_0^6 \frac{e^{\sqrt{(6-x)/(6+x)}}\,dx}{(6+x)\sqrt{36-x^2}}.$

11.28. $\displaystyle\int_1^{64} \frac{6-\sqrt{x}+\sqrt[4]{x}}{\sqrt{x^3}-7x-6\sqrt[4]{x^3}}\,dx.$

11.29. $\displaystyle\int_0^1 \frac{\left(4\sqrt{1-x}-\sqrt{x+1}\right)dx}{\left(\sqrt{x+1}+4\sqrt{1-x}\right)(x+1)^2}.$

11.30. $\displaystyle\int_0^3 \frac{e^{\sqrt{(3-x)/(3+x)}}\,dx}{(3+x)\sqrt{9-x^2}}.$

11.31. $\displaystyle\int_0^2 \frac{\left(4\sqrt{2-x}-\sqrt{x+2}\right)dx}{\left(\sqrt{x+2}+4\sqrt{2-x}\right)(x+2)^2}.$

Задача 12. Вычислить определенные интегралы.

12.1. $\displaystyle\int_0^{16} \sqrt{256-x^2}\,dx.$

12.2. $\displaystyle\int_0^1 x^2\sqrt{1-x^2}\,dx.$

12.3. $\displaystyle\int_0^5 \frac{dx}{\left(25+x^2\right)\sqrt{25+x^2}}.$

12.4. $\displaystyle\int_0^3 \frac{dx}{\left(9+x^2\right)^{3/2}}.$

12.5. $\displaystyle\int_0^{\sqrt5/2} \frac{dx}{\sqrt{\left(5-x^2\right)^3}}.$

12.6. $\displaystyle\int_1^2 \frac{\sqrt{x^2-1}}{x^4}\,dx.$

12.7. $\displaystyle\int_0^{\sqrt2/2} \frac{x^4\,dx}{\sqrt{\left(1-x^2\right)^3}}.$

12.8. $\displaystyle\int_0^{\sqrt3} \frac{dx}{\sqrt{\left(4-x^2\right)^3}}.$

12.9. $\displaystyle\int_0^1 \frac{x^4 dx}{\left(2-x^2\right)^{3/2}}.$

12.10. $\displaystyle\int_0^2 \frac{x^2 dx}{\sqrt{16-x^2}}.$

12.11. $\displaystyle\int_0^2 \sqrt{4-x^2}\, dx.$

12.12. $\displaystyle\int_0^4 \frac{dx}{\left(16+x^2\right)^{3/2}}.$

12.13. $\displaystyle\int_0^4 x^2\sqrt{16-x^2}\, dx.$

12.14. $\displaystyle\int_0^{5/2} \frac{x^2 dx}{\sqrt{25-x^2}}.$

12.15. $\displaystyle\int_0^5 x^2\sqrt{25-x^2}\, dx.$

12.16. $\displaystyle\int_0^4 \sqrt{16-x^2}\, dx.$

12.17. $\displaystyle\int_0^{4\sqrt{3}} \frac{dx}{\sqrt{\left(64-x^2\right)^3}}.$

12.18. $\displaystyle\int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2-2}}{x^4}\, dx.$

12.19. $\displaystyle\int_0^{2\sqrt{2}} \frac{x^4 dx}{\left(16-x^2\right)\sqrt{16-x^2}}.$

12.20. $\displaystyle\int_{-3}^3 x^2\sqrt{9-x^2}\, dx.$

12.21. $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{\sqrt{\left(1+x^2\right)^3}}.$

12.22. $\displaystyle\int_0^2 \frac{dx}{\sqrt{\left(16-x^2\right)^3}}.$

12.23. $\displaystyle\int_0^2 \frac{x^4 dx}{\sqrt{\left(8-x^2\right)^3}}.$

12.24. $\displaystyle\int_3^6 \frac{\sqrt{x^2-9}}{x^4}\, dx.$

12.25. $\displaystyle\int_0^1 \sqrt{4-x^2}\, dx.$

12.26. $\displaystyle\int_2^4 \frac{\sqrt{x^2-4}}{x^4}\, dx.$

12.27. $\displaystyle\int_0^2 \frac{dx}{\left(4+x^2\right)\sqrt{4+x^2}}.$

12.28. $\displaystyle\int_0^{\sqrt{2}} \frac{x^4 dx}{\left(4-x^2\right)^{3/2}}.$

12.29. $\displaystyle\int_0^{1/\sqrt{2}} \frac{dx}{\left(1-x^2\right)\sqrt{1-x^2}}.$

12.30. $\displaystyle\int_0^1 \frac{x^2 dx}{\sqrt{4-x^2}}.$

12.31. $\displaystyle\int_0^{3/2} \frac{x^2 dx}{\sqrt{9-x^2}}.$

Задача 13. Найти неопределенные интегралы.

13.1. $\int \dfrac{\sqrt{1+\sqrt{x}}}{x\sqrt[4]{x^3}}\,dx.$

13.2. $\int \dfrac{\sqrt[3]{1+\sqrt{x}}}{x\sqrt[3]{x^2}}\,dx.$

13.3. $\int \dfrac{\sqrt{1+\sqrt[3]{x}}}{x\sqrt{x}}\,dx.$

13.4. $\int \dfrac{\sqrt[3]{1+\sqrt[3]{x}}}{x\sqrt[9]{x^4}}\,dx.$

13.5. $\int \dfrac{\sqrt[3]{1+\sqrt[3]{x^2}}}{x\sqrt[9]{x^8}}\,dx.$

13.6. $\int \dfrac{\sqrt[3]{\left(1+\sqrt[3]{x}\right)^2}}{x\sqrt[9]{x^5}}\,dx.$

13.7. $\int \dfrac{\sqrt[3]{\left(1+\sqrt[3]{x^2}\right)^2}}{x^2\,\sqrt[9]{x}}\,dx.$

13.8. $\int \dfrac{\sqrt[3]{\left(1+\sqrt{x}\right)^2}}{x^6\sqrt{x^5}}\,dx.$

13.9. $\int \dfrac{\sqrt{1+\sqrt[3]{x^2}}}{x^2}\,dx.$

13.10. $\int \dfrac{\sqrt{1+x}}{x^2\sqrt{x}}\,dx.$

13.11. $\int \dfrac{\sqrt[4]{\left(1+\sqrt{x}\right)^3}}{x\sqrt[8]{x^7}}\,dx.$

13.12. $\int \dfrac{\sqrt[4]{\left(1+\sqrt[3]{x}\right)^3}}{x\sqrt[12]{x^7}}\,dx.$

13.13. $\int \dfrac{\sqrt[4]{\left(1+\sqrt[3]{x^2}\right)^3}}{x^2\,\sqrt[6]{x}}\,dx.$

13.14. $\int \dfrac{\sqrt{1+\sqrt[4]{x^3}}}{x^2\,\sqrt[8]{x}}\,dx.$

13.15. $\int \dfrac{\sqrt[3]{1+\sqrt[4]{x^3}}}{x^2}\,dx.$

13.16. $\int \dfrac{\sqrt[3]{\left(1+\sqrt[4]{x^3}\right)^2}}{x^2\,\sqrt[4]{x}}\,dx.$

13.17. $\int \dfrac{\sqrt[5]{\left(1+\sqrt{x}\right)^4}}{x\sqrt[10]{x^9}}\,dx.$

13.18. $\int \dfrac{\sqrt[5]{\left(1+\sqrt[3]{x}\right)^4}}{x\sqrt[5]{x^3}}\,dx.$

13.19. $\int \dfrac{\sqrt[5]{\left(1+\sqrt[3]{x^2}\right)^4}}{x^2\,\sqrt[5]{x}}\,dx.$

13.20. $\int \dfrac{\sqrt[5]{\left(1+\sqrt[4]{x^3}\right)^4}}{x^2\,\sqrt[20]{x^7}}\,dx.$

13.21. $\int \dfrac{\sqrt[5]{1+\sqrt[5]{x^4}}}{x^2\,\sqrt[25]{x^{11}}}\,dx.$

13.22. $\int \dfrac{\sqrt{1+\sqrt[5]{x^4}}}{x^2\,\sqrt[5]{x}}\,dx.$

13.23. $\displaystyle\int \frac{\sqrt[3]{1+\sqrt[5]{x^4}}}{x^2\sqrt[15]{x}}\,dx.$

13.24. $\displaystyle\int \frac{\sqrt[3]{\left(1+\sqrt[5]{x^4}\right)^2}}{x^2\sqrt[3]{x}}\,dx.$

13.25. $\displaystyle\int \frac{\sqrt[4]{\left(1+\sqrt[5]{x^4}\right)^3}}{x^2\sqrt[5]{x^2}}\,dx.$

13.26. $\displaystyle\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{x\sqrt[3]{x}}\,dx.$

13.27. $\displaystyle\int \frac{\sqrt[3]{\left(1+\sqrt[4]{x}\right)^2}}{x\sqrt[12]{x^5}}\,dx.$

13.28. $\displaystyle\int \frac{\sqrt[4]{1+\sqrt[3]{x}}}{x\sqrt[12]{x^5}}\,dx.$

13.29. $\displaystyle\int \frac{\sqrt[4]{1+\sqrt[3]{x^2}}}{x\sqrt[6]{x^5}}\,dx.$

13.30. $\displaystyle\int \frac{\sqrt[3]{1+\sqrt[5]{x}}}{x\sqrt[15]{x^4}}\,dx.$

13.31. $\displaystyle\int \frac{\sqrt[5]{1+\sqrt[3]{x}}}{x\sqrt[5]{x^2}}\,dx.$

Задача 14. Вычислить площади фигур, ограниченных графиками функций.

14.1. $y=(x-2)^3,$
$y=4x-8.$

14.2. $y=x\sqrt{9-x^2},\quad y=0,$
$(0\le x\le 3).$

14.3. $y=4-x^2,$
$y=x^2-2x.$

14.4. $y=\sin x\cos^2 x,\quad y=0,$
$(0\le x\le \pi/2).$

14.5. $y=\sqrt{4-x^2},\quad y=0,$
$x=0,\quad x=1.$

14.6. $y=x^2\sqrt{4-x^2},\quad y=0,$
$(0\le x\le 2).$

14.7. $y=\cos x\sin^2 x,\quad y=0,$
$(0\le x\le \pi/2).$

14.8. $y=\sqrt{e^x-1},\quad y=0,$
$x=\ln 2.$

14.9. $y=\dfrac{1}{x\sqrt{1+\ln x}},\quad y=0,$
$x=1,\quad x=e^3.$

14.10. $y=\arccos x,\quad y=0,$
$x=0.$

14.11. $y=(x+1)^2,$
$y^2=x+1.$

14.12. $y=2x-x^2+3,$
$y=x^2-4x+3.$

14.13. $y = x\sqrt{36 - x^2}, \quad y = 0,$
$(0 \le x \le 6).$

14.14. $x = \arccos y, \quad x = 0,$
$y = 0.$

14.15. $y = \operatorname{arctg} x, \quad y = 0,$
$x = \sqrt{3}.$

14.16. $y = x^2\sqrt{8 - x^2}, \quad y = 0,$
$\left(0 \le x \le 2\sqrt{2}\right).$

14.17. $x = \sqrt{e^y - 1}, \quad x = 0,$
$y = \ln 2.$

14.18. $y = x\sqrt{4 - x^2}, \quad y = 0,$
$(0 \le x \le 2).$

14.19. $y = \dfrac{x}{1 + \sqrt{x}}, \quad y = 0,$
$x = 1.$

14.20. $y = \dfrac{1}{1 + \cos x}, \quad y = 0,$
$x = \pi/2, \quad x = -\pi/2.$

14.21. $x = (y - 2)^3,$
$x = 4y - 8.$

14.22. $y = \cos^5 x \sin 2x, \quad y = 0,$
$(0 \le x \le \pi/2).$

14.23. $y = \dfrac{x}{\left(x^2 + 1\right)^2}, \quad y = 0,$
$x = 1.$

14.24. $x = 4 - y^2,$
$x = y^2 - 2y.$

14.25. $x = \dfrac{1}{y\sqrt{1 + \ln y}}, \quad x = 0,$
$y = 1, \quad y = e^3.$

14.26. $y = \dfrac{e^{1/x}}{x^2}, \quad y = 0,$
$x = 2, \quad x = 1.$

14.27. $y = x^2\sqrt{16 - x^2}, \quad y = 0,$
$(0 \le x \le 4).$

14.28. $x = \sqrt{4 - y^2}, \quad x = 0,$
$y = 0, \quad y = 1.$

14.29. $y = (x - 1)^2,$
$y^2 = x - 1.$

14.30. $y = x^2 \cos x, \quad y = 0,$
$(0 \le x \le \pi/2).$

14.31. $x = 4 - (y - 1)^2,$
$x = y^2 - 4y + 3.$

Задача 15. Вычислить площади фигур, ограниченных линиями, заданными уравнениями.

15.1. $\begin{cases} x = 4\sqrt{2}\cos^3 t, \\ y = 2\sqrt{2}\sin^3 t, \end{cases}$
$x = 2 \ (x \geq 2).$

15.2. $\begin{cases} x = \sqrt{2}\cos t, \\ y = 2\sqrt{2}\sin t, \end{cases}$
$y = 2 \ (y \geq 2).$

15.3. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \end{cases}$
$y = 4 \ (0 < x < 8\pi, \ y \geq 4).$

15.4. $\begin{cases} x = 16\cos^3 t, \\ y = 2\sin^3 t, \end{cases}$
$x = 2 \ (x \geq 2).$

15.5. $\begin{cases} x = 2\cos t, \\ y = 6\sin t, \end{cases}$
$y = 3 \ (y \geq 3).$

15.6. $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \end{cases}$
$y = 3 \ (0 < x < 4\pi, \ y \geq 3).$

15.7. $\begin{cases} x = 16\cos^3 t, \\ y = \sin^3 t, \end{cases}$
$x = 6\sqrt{3} \ \left(x \geq 6\sqrt{3}\right).$

15.8. $\begin{cases} x = 6\cos t, \\ y = 2\sin t, \end{cases}$
$y = \sqrt{3} \ \left(y \geq \sqrt{3}\right).$

15.9. $\begin{cases} x = 3(t - \sin t), \\ y = 3(1 - \cos t), \end{cases}$
$y = 3 \ (0 < x < 6\pi, \ y \geq 3).$

15.10. $\begin{cases} x = 8\sqrt{2}\cos^3 t, \\ y = \sqrt{2}\sin^3 t, \end{cases}$
$x = 4 \ (x \geq 4).$

15.11. $\begin{cases} x = 2\sqrt{2}\cos t, \\ y = 3\sqrt{2}\sin t, \end{cases}$
$y = 3 \ (y \geq 3).$

15.12. $\begin{cases} x = 6(t - \sin t), \\ y = 6(1 - \cos t), \end{cases}$
$y = 9 \ (0 < x < 12\pi, \ y \geq 9).$

15.13. $\begin{cases} x = 32\cos^3 t, \\ y = \sin^3 t, \end{cases}$
$x = 4 \ (x \geq 4).$

15.14. $\begin{cases} x = 3\cos t, \\ y = 8\sin t, \end{cases}$
$y = 4 \ (y \geq 4).$

15.15. $\begin{cases} x = 6(t - \sin t), \\ y = 6(1 - \cos t), \end{cases}$
$y = 6 \ (0 < x < 12\pi, \ y \geq 6).$

15.16. $\begin{cases} x = 8\cos^3 t, \\ y = 4\sin^3 t, \end{cases}$
$x = 3\sqrt{3} \ \left(x \geq 3\sqrt{3}\right).$

15.17. $\begin{cases} x = 6\cos^3 t, \\ y = 4\sin^3 t, \end{cases}$
$x = 2\sqrt{3} \ \left(x \geq 2\sqrt{3} \right).$

15.18. $\begin{cases} x = 10(t - \sin t), \\ y = 10(1 - \cos t), \end{cases}$
$y = 15 \ \left(0 < x < 20\pi, \ y \geq 15 \right).$

15.19. $\begin{cases} x = 2\sqrt{2}\cos^3 t, \\ y = \sqrt{2}\sin^3 t, \end{cases}$
$x = 1 \ (x \geq 1).$

15.20. $\begin{cases} x = \sqrt{2}\cos t, \\ y = 4\sqrt{2}\sin t, \end{cases}$
$y = 4 \ (y \geq 4).$

15.21. $\begin{cases} x = t - \sin t, \\ y = 1 - \cos t, \end{cases}$
$y = 1 \ (0 < x < 2\pi, \ y \geq 1).$

15.22. $\begin{cases} x = 8\cos^3 t, \\ y = 8\sin^3 t, \end{cases}$
$x = 1 \ (x \geq 1).$

15.23. $\begin{cases} x = 9\cos t, \\ y = 4\sin t, \end{cases}$
$y = 2 \ (y \geq 2).$

15.24. $\begin{cases} x = 8(t - \sin t), \\ y = 8(1 - \cos t), \end{cases}$
$y = 12 \ (0 < x < 16\pi, \ y \geq 12).$

15.25. $\begin{cases} x = 24\cos^3 t, \\ y = 2\sin^3 t, \end{cases}$
$x = 9\sqrt{3} \ \left(x \geq 9\sqrt{3} \right).$

15.26. $\begin{cases} x = 3\cos t, \\ y = 8\sin t, \end{cases}$
$y = 4\sqrt{3} \ \left(y \geq 4\sqrt{3} \right).$

15.27. $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \end{cases}$
$y = 2 \ (0 < x < 4\pi, \ y \geq 2).$

15.28. $\begin{cases} x = 4\sqrt{2}\cos^3 t, \\ y = \sqrt{2}\sin^3 t, \end{cases}$
$x = 2 \ (x \geq 2).$

15.29. $\begin{cases} x = 2\sqrt{2}\cos t, \\ y = 5\sqrt{2}\sin t, \end{cases}$
$y = 5 \ (y \geq 5).$

15.30. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \end{cases}$
$y = 6 \ (0 < x < 8\pi, \ y \geq 6).$

15.31. $\begin{cases} x = 32\cos^3 t, \\ y = 3\sin^3 t, \end{cases}$
$x = 12\sqrt{3} \ \left(x \geq 12\sqrt{3} \right).$

Задача 16. Вычислить площади фигур, ограниченных линиями, заданными в полярных координатах.

16.1. $r = 4\cos 3\varphi, \quad r = 2 \quad (r \geq 2).$

16.2. $r = \cos 2\varphi.$

16.3. $\begin{array}{l} r = \sqrt{3}\cos\varphi, \quad r = \sin\varphi, \\ \quad (0 \leq \varphi \leq \pi/2). \end{array}$

16.4. $r = 4\sin 3\varphi, \quad r = 2 \quad (r \geq 2).$

16.5. $\begin{array}{l} r = 2\cos\varphi, \quad r = 2\sqrt{3}\sin\varphi, \\ \quad (0 \leq \varphi \leq \pi/2). \end{array}$

16.6. $r = \sin 3\varphi.$

16.7. $r = 6\sin 3\varphi, \quad r = 3 \quad (r \geq 3).$

16.8. $r = \cos 3\varphi.$

16.9. $\begin{array}{l} r = \cos\varphi, \\ r = \sqrt{2}\sin(\varphi - \pi/4), \\ (-\pi/4 \leq \varphi \leq \pi/2). \end{array}$

16.10. $\begin{array}{l} r = \sin\varphi, \\ r = \sqrt{2}\cos(\varphi - \pi/4), \\ (0 \leq \varphi \leq 3\pi/4). \end{array}$

16.11. $r = 6\cos 3\varphi, \quad r = 3 \quad (r \geq 3).$

16.12. $r = 1/2 + \sin\varphi.$

16.13. $\begin{array}{l} r = \cos\varphi, \quad r = \sin\varphi, \\ (0 \leq \varphi \leq \pi/2). \end{array}$

16.14. $\begin{array}{l} r = \sqrt{2}\cos(\varphi - \pi/4), \\ r = \sqrt{2}\sin(\varphi - \pi/4), \\ (\pi/4 \leq \varphi \leq 3\pi/4). \end{array}$

16.15. $r = \cos\varphi, \quad r = 2\cos\varphi.$

16.16. $r = \sin\varphi, \quad r = 2\sin\varphi.$

16.17. $r = 1 + \sqrt{2}\cos\varphi.$

16.18. $r = 1/2 + \cos\varphi.$

16.19. $r = 1 + \sqrt{2}\sin\varphi.$

16.20. $r = (5/2)\sin\varphi, \quad r = (3/2)\sin\varphi.$

16.21. $r = (3/2)\cos\varphi, \quad r = (5/2)\cos\varphi.$

16.22. $r = 4\cos 4\varphi.$

16.23. $r = \sin 6\varphi.$

16.24. $r = 2\cos\varphi, \quad r = 3\cos\varphi.$

16.25. $r = \cos\varphi + \sin\varphi.$

16.26. $r = 2\sin 4\varphi.$

16.27. $r = 2\cos 6\varphi.$

16.28. $r = \cos\varphi - \sin\varphi.$

16.29. $r = 3\sin\varphi, \quad r = 5\sin\varphi.$ 16.30. $r = 2\sin\varphi, \quad r = 4\sin\varphi.$

16.31. $r = 6\sin\varphi, \quad r = 4\sin\varphi.$

Задача 17. Вычислить длины дуг кривых, заданных уравнениями в прямоугольной системе координат.

17.1. $y = \ln x, \quad \sqrt{3} \le x \le \sqrt{15}.$ 17.2. $y = \dfrac{x^2}{4} - \dfrac{\ln x}{2}, \quad 1 \le x \le 2.$

17.3. $y = \sqrt{1 - x^2} + \arcsin x, \quad 0 \le x \le 7/9.$ 17.3. $y = \ln\dfrac{5}{2x}, \quad \sqrt{3} \le x \le \sqrt{8}.$

17.5. $y = -\ln\cos x, \quad 0 \le x \le \pi/6.$ 17.6. $y = e^x + 6, \quad \ln\sqrt{8} \le x \le \ln\sqrt{15}.$

17.7. $y = 2 + \arcsin\sqrt{x} + \sqrt{x - x^2}, \quad 1/4 \le x \le 1.$

17.8. $y = \ln\left(x^2 - 1\right), \quad 2 \le x \le 3.$

17.9. $y = \sqrt{1 - x^2} + \arccos x, \quad 0 \le x \le 8/9.$ 17.10. $y = \ln\left(1 - x^2\right), \quad 0 \le x \le 1/4.$

17.11. $y = 2 + \operatorname{ch} x, \quad 0 \le x \le 1.$ 17.12. $y = 1 - \ln\cos x, \quad 0 \le x \le \pi/6.$

17.13. $y = e^x + 13, \quad \ln\sqrt{15} \le x \le \ln\sqrt{24}.$

17.14. $y = -\arccos\sqrt{x} + \sqrt{x - x^2}, \quad 0 \le x \le 1/4.$

17.15. $y = 2 - e^x, \quad \ln\sqrt{3} \le x \le \ln\sqrt{8}.$

17.16. $y = \arcsin x - \sqrt{1 - x^2}, \quad 0 \le x \le 15/16.$

17.17. $y = 1 - \ln\sin x, \quad \pi/3 \le x \le \pi/2.$ 17.18. $y = 1 - \ln\left(x^2 - 1\right), \quad 3 \le x \le 4.$

17.19. $y = \sqrt{x - x^2} - \arccos\sqrt{x} + 5, \quad 1/9 \le x \le 1.$

17.20. $y = -\arccos x + \sqrt{1 - x^2} + 1, \quad 0 \le x \le 9/16.$

17.21. $y = \ln\sin x, \quad \pi/3 \le x \le \pi/2.$ 17.22. $y = \ln 7 - \ln x, \quad \sqrt{3} \le x \le \sqrt{8}.$

17.23. $y = \operatorname{ch} x + 3, \quad 0 \le x \le 1.$

17.24. $y = 1 + \arcsin x - \sqrt{1 - x^2}, \quad 0 \le x \le 3/4.$

17.25. $y = \ln\cos x + 2, \quad 0 \le x \le \pi/6.$

17.26. $y = \mathrm{e}^x + 26, \quad \ln\sqrt{8} \le x \le \ln\sqrt{24}.$ 　　　 17.27. $y = \dfrac{\mathrm{e}^x + \mathrm{e}^{-x}}{2} + 3, \quad 0 \le x \le 2.$

17.28. $y = \arccos\sqrt{x} - \sqrt{x - x^2} + 4, \quad 0 \le x \le 1/2.$

17.29. $y = \dfrac{\mathrm{e}^x + \mathrm{e}^{-x} + 3}{4}, \quad 0 \le x \le 2.$ 　　　 17.30. $y = \mathrm{e}^x + \mathrm{e}, \quad \ln\sqrt{3} \le x \le \ln\sqrt{15}.$

17.31. $y = \dfrac{1 - \mathrm{e}^x - \mathrm{e}^{-x}}{2}, \quad 0 \le x \le 3.$

Задача 18. Вычислить длины дуг кривых, заданных параметрическими уравнениями.

18.1. $\begin{cases} x = 5(t - \sin t), \\ y = 5(1 - \cos t), \\ \quad 0 \le t \le \pi. \end{cases}$ 　　　 18.2. $\begin{cases} x = 3(2\cos t - \cos 2t), \\ y = 3(2\sin t - \sin 2t), \\ \quad 0 \le t \le 2\pi. \end{cases}$

18.3. $\begin{cases} x = 4(\cos t + t\sin t), \\ y = 4(\sin t - t\cos t), \\ \quad 0 \le t \le 2\pi. \end{cases}$ 　　　 18.4. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \\ \quad 0 \le t \le \pi. \end{cases}$

18.5. $\begin{cases} x = 10\cos^3 t, \\ y = 10\sin^3 t, \\ \quad 0 \le t \le \pi/2. \end{cases}$ 　　　 18.6. $\begin{cases} x = \mathrm{e}^t(\cos t + \sin t), \\ y = \mathrm{e}^t(\cos t - \sin t), \\ \quad 0 \le t \le \pi. \end{cases}$

18.7. $\begin{cases} x = 3(t - \sin t), \\ y = 3(1 - \cos t), \\ \quad \pi \le t \le 2\pi. \end{cases}$ 　　　 18.8. $\begin{cases} x = \dfrac{1}{2}\cos t - \dfrac{1}{4}\cos 2t, \\ y = \dfrac{1}{2}\sin t - \dfrac{1}{4}\sin 2t, \\ \quad \pi/2 \le t \le 2\pi/3. \end{cases}$

18.9. $\begin{cases} x = 3(\cos t + t\sin t), \\ y = 3(\sin t - t\cos t), \\ \quad 0 \le t \le \pi/3. \end{cases}$ 　　　 18.10. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \\ \quad 0 \le t \le \pi/3. \end{cases}$

18.11. $\begin{cases} x = 6\cos^3 t, \\ y = 6\sin^3 t, \\ 0 \le t \le \pi/3. \end{cases}$

18.12. $\begin{cases} x = e^t(\cos t + \sin t), \\ y = e^t(\cos t - \sin t), \\ \pi/2 \le t \le \pi. \end{cases}$

18.13. $\begin{cases} x = 2{,}5(t - \sin t), \\ y = 2{,}5(1 - \cos t), \\ \pi/2 \le t \le \pi. \end{cases}$

18.14. $\begin{cases} x = 3{,}5(2\cos t - \cos 2t), \\ y = 3{,}5(2\sin t - \sin 2t), \\ 0 \le t \le \pi/2. \end{cases}$

18.15. $\begin{cases} x = 6(\cos t + t\sin t), \\ y = 6(\sin t - t\cos t), \\ 0 \le t \le \pi. \end{cases}$

18.16. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \\ 0 \le t \le \pi/2. \end{cases}$

18.17. $\begin{cases} x = 8\cos^3 t, \\ y = 8\sin^3 t, \\ 0 \le t \le \pi/6. \end{cases}$

18.18. $\begin{cases} x = e^t(\cos t + \sin t), \\ y = e^t(\cos t - \sin t), \\ 0 \le t \le 2\pi. \end{cases}$

18.19. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \\ \pi/2 \le t \le 2\pi/3. \end{cases}$

18.20. $\begin{cases} x = 2(2\cos t - \cos 2t), \\ y = 2(2\sin t - \sin 2t), \\ 0 \le t \le \pi/3. \end{cases}$

18.21. $\begin{cases} x = 8(\cos t + t\sin t), \\ y = 8(\sin t - t\cos t), \\ 0 \le t \le \pi/4. \end{cases}$

18.22. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \\ 0 \le t \le 2\pi. \end{cases}$

18.23. $\begin{cases} x = 4\cos^3 t, \\ y = 4\sin^3 t, \\ \pi/6 \le t \le \pi/4. \end{cases}$

18.24. $\begin{cases} x = e^t(\cos t + \sin t), \\ y = e^t(\cos t - \sin t), \\ 0 \le t \le 3\pi/2. \end{cases}$

18.25. $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \\ 0 \le t \le \pi/2. \end{cases}$

18.26. $\begin{cases} x = 4(2\cos t - \cos 2t), \\ y = 4(2\sin t - \sin 2t), \\ 0 \le t \le \pi. \end{cases}$

18.27. $\begin{cases} x = 2(\cos t + t\sin t), \\ y = 2(\sin t - t\cos t), \\ 0 \le t \le \pi/2. \end{cases}$

18.28. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \\ 0 \le t \le 3\pi. \end{cases}$

18.29. $\begin{cases} x = 2\cos^3 t, \\ y = 2\sin^3 t, \end{cases}$
$0 \le t \le \pi/4.$

18.30. $\begin{cases} x = e^t (\cos t + \sin t), \\ y = e^t (\cos t - \sin t), \end{cases}$
$\pi/6 \le t \le \pi/4.$

18.31. $\begin{cases} x = (t^2 - 2)\sin t + 2t\cos t, \\ y = (2 - t^2)\cos t + 2t\sin t, \end{cases}$
$0 \le t \le \pi.$

Задача 19. Вычислить длины дуг кривых, заданных уравнениями в полярных координатах.

19.1. $\rho = 3e^{3\varphi/4}, \quad -\pi/2 \le \varphi \le \pi/2.$

19.2. $\rho = 2e^{4\varphi/3}, \quad -\pi/2 \le \varphi \le \pi/2.$

19.3. $\rho = \sqrt{2}\,e^{\varphi}, \quad -\pi/2 \le \varphi \le \pi/2.$

19.4. $\rho = 5e^{5\varphi/12}, \quad -\pi/2 \le \varphi \le \pi/2.$

19.5. $\rho = 6e^{12\varphi/5}, \quad -\pi/2 \le \varphi \le \pi/2.$

19.6. $\rho = 3e^{3\varphi/4}, \quad 0 \le \varphi \le \pi/3.$

19.7. $\rho = 4e^{4\varphi/3}, \quad 0 \le \varphi \le \pi/3.$

19.8. $\rho = \sqrt{2}\,e^{\varphi}, \quad 0 \le \varphi \le \pi/3.$

19.9. $\rho = 5e^{5\varphi/12}, \quad 0 \le \varphi \le \pi/3.$

19.10. $\rho = 12e^{12\varphi/5}, \quad 0 \le \varphi \le \pi/3.$

19.11. $\rho = 1 - \sin\varphi, \quad -\pi/2 \le \varphi \le -\pi/6.$

19.12.

$\rho = 2(1 - \cos\varphi), \quad -\pi \le \varphi \le -\pi/2.$

19.13. $\rho = 3(1 + \sin\varphi), \quad -\pi/6 \le \varphi \le 0.$

19.14. $\rho = 4(1 - \sin\varphi), \quad 0 \le \varphi \le \pi/6.$

19.15. $\rho = 5(1 - \cos\varphi), \quad -\pi/3 \le \varphi \le 0.$

19.16. $\rho = 6(1 + \sin\varphi), \quad -\pi/2 \le \varphi \le 0.$

19.17. $\rho = 7(1 - \sin\varphi), \quad -\pi/6 \le \varphi \le \pi/6.$

19.18. $\rho = 8(1 - \cos\varphi), \quad -2\pi/3 \le \varphi \le 0.$

19.19. $\rho = 2\varphi, \quad 0 \le \varphi \le 3/4.$

19.20. $\rho = 2\varphi, \quad 0 \le \varphi \le 4/3.$

19.21. $\rho = 2\varphi, \quad 0 \le \varphi \le 5/12.$

19.22. $\rho = 2\varphi, \quad 0 \le \varphi \le 12/5.$

19.23. $\rho = 4\varphi, \quad 0 \le \varphi \le 3/4.$

19.24. $\rho = 3\varphi, \quad 0 \le \varphi \le 4/3.$

19.25. $\rho = 5\varphi, \quad 0 \le \varphi \le 12/5.$

19.26. $\rho = 2\cos\varphi, \quad 0 \le \varphi \le \pi/6.$

19.27. $\rho = 8\cos\varphi, \quad 0 \le \varphi \le \pi/4.$

19.28. $\rho = 6\cos\varphi, \quad 0 \le \varphi \le \pi/3.$

19.29. $\rho = 2\sin\varphi, \quad 0 \le \varphi \le \pi/6.$

19.30. $\rho = 8\sin\varphi, \quad 0 \le \varphi \le \pi/4.$

19.31. $\rho = 6\sin\varphi, \quad 0 \le \varphi \le \pi/3.$

Задача 20. Вычислить объемы тел, ограниченных поверхностями.

20.1. $\dfrac{x^2}{9} + y^2 = 1, \ z = y, \ z = 0 \ (y \ge 0).$ 20.2. $z = x^2 + 4y^2, \ z = 2.$

20.3. $\dfrac{x^2}{9} + \dfrac{y^2}{4} - z^2 = 1, \ z = 0, \ z = 3.$ 20.4. $\dfrac{x^2}{9} + \dfrac{y^2}{4} - \dfrac{z^2}{36} = -1, \ z = 12.$

20.5. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1, \ z = 1, \ z = 0.$

20.6. $x^2 + y^2 = 9, \ z = y, \ z = 0 \ (y \ge 0).$

20.7. $z = x^2 + 9y^2, \ z = 3.$ 20.8. $\dfrac{x^2}{4} + y^2 - z^2 = 1, \ z = 0, \ z = 3.$

20.9. $\dfrac{x^2}{9} + \dfrac{y^2}{16} - \dfrac{z^2}{64} = -1, \ z = 16.$ 20.10. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + \dfrac{z^2}{16} = 1, \ z = 2, \ z = 0.$

20.11. $\dfrac{x^2}{3} + \dfrac{y^2}{4} = 1, \ z = y\sqrt{3}, \ z = 0 \ (y \ge 0).$ 20.12. $z = 2x^2 + 8y^2, \ z = 4.$

20.13. $\dfrac{x^2}{81} + \dfrac{y^2}{25} - z^2 = 1, \ z = 0, \ z = 2.$ 20.14. $\dfrac{x^2}{4} + \dfrac{y^2}{9} - \dfrac{z^2}{36} = -1, \ z = 12.$

20.15. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + \dfrac{z^2}{36} = 1, \ z = 3, \ z = 0.$

20.16. $\dfrac{x^2}{3} + \dfrac{y^2}{16} = 1, \ z = y\sqrt{3}, \ z = 0 \ (y \ge 0).$

20.17. $z = x^2 + 5y^2, \ z = 5.$ 20.18. $\dfrac{x^2}{9} + \dfrac{y^2}{4} - z^2 = 1, \ z = 0, \ z = 4.$

20.19. $\dfrac{x^2}{9} + \dfrac{y^2}{25} - \dfrac{z^2}{100} = -1, \ z = 20.$ 20.20. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + \dfrac{z^2}{64} = 1, \ z = 4, \ z = 0.$

20.21. $\dfrac{x^2}{27} + \dfrac{y^2}{25} = 1, \ z = \dfrac{y}{\sqrt{3}}, \ z = 0 \ (y \ge 0).$ 20.22. $z = 4x^2 + 9y^2, \ z = 6.$

20.23. $x^2 + \dfrac{y^2}{4} - z^2 = 1, \ z = 0, \ z = 3.$ 20.24. $\dfrac{x^2}{25} + \dfrac{y^2}{9} - \dfrac{z^2}{100} = -1, \ z = 20.$

20.25. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + \dfrac{z^2}{100} = 1, \ z = 5, \ z = 0.$

20.26. $\dfrac{x^2}{27} + y^2 = 1, \ z = \dfrac{y}{\sqrt{3}}, \ z = 0 \ (y \geq 0).$

20.27. $z = 2x^2 + 18y^2, \ z = 6.$

20.28. $\dfrac{x^2}{25} + \dfrac{y^2}{9} - z^2 = 1, \ z = 0, \ z = 2.$

20.29. $\dfrac{x^2}{16} + \dfrac{y^2}{9} - \dfrac{z^2}{64} = -1, \ z = 16.$

20.30. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + \dfrac{z^2}{144} = 1, \ z = 6, \ z = 0.$

20.31. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + \dfrac{z^2}{196} = 1, \ z = 7, \ z = 0.$

Задача 21. Вычислить объемы тел, образованных вращением фигур, ограниченных графиками функций. В вариантах 1–16 ось вращения Ox, в вариантах 17–31 ось вращения Oy.

21.1. $y = -x^2 + 5x - 6, \ y = 0.$

21.2. $2x - x^2 - y = 0, \ 2x^2 - 4x + y = 0.$

21.3. $y = 3\sin x, \ y = \sin x, \ 0 \leq x \leq \pi.$

21.4. $y = 5\cos x, \ y = \cos x, \ x = 0, \ x \geq 0.$

21.5. $y = \sin^2 x, \ x = \pi/2, \ y = 0.$

21.6. $x = \sqrt[3]{y - 2}, \ x = 1, \ y = 1.$

21.7. $y = xe^x, \ y = 0, \ x = 1.$

21.8. $y = 2x - x^2, \ y = -x + 2, \ x = 0.$

21.9. $y = 2x - x^2, \ y = -x + 2.$

21.10. $y = e^{1-x}, \ y = 0, \ x = 0, \ x = 1.$

21.11. $y = x^2, \ y^2 - x = 0.$

21.12. $x^2 + (y - 2)^2 = 1.$

21.13. $y = 1 - x^2, \ x = 0, \ x = \sqrt{y - 2}, \ x = 1.$

21.14. $y = x^2, \ y = 1, \ x = 2.$

21.15. $y = x^3, \ y = \sqrt{x}.$

21.16. $y = \sin(\pi x/2), \ y = x^2.$

21.17. $y = \arccos(x/3), \ y = \arccos x, \ y = 0.$

21.18. $y = \arcsin(x/5), \ y = \arcsin x, \ y = \pi/2.$

21.19. $y = x^2, \ x = 2, \ y = 0.$

21.20. $y = x^2 + 1, \ y = x, \ x = 0, \ y = 0.$

21.21. $y = \sqrt{x - 1}, \ y = 0, \ y = 1, \ x = 0{,}5.$

21.22. $y = \ln x, \ x = 2, \ y = 0.$

21.23. $y = (x-1)^2$, $y = 1$.   21.24.

$y^2 = x - 2$, $y = 0$, $y = x^3$, $y = 1$.

21.25. $y = x^3$, $y = x^2$.   21.26. $y = \arccos(x/5)$, $y = \arccos(x/3)$, $y = 0$.

21.27. $y = \arcsin x$, $y = \arccos x$, $y = 0$.   21.28. $y = x^2 - 2x + 1$, $x = 2$, $y = 0$.

21.29. $y = x^3$, $y = x$.   21.30.

$y = \arccos x$, $y = \arcsin x$, $x = 0$.

21.31. $y = (x-1)^2$, $x = 0$, $x = 2$, $y = 0$.

Задача 22

Варианты 1–10

Вычислить силу, с которой вода давит на плотину, сечение которой имеет форму равнобочной трапеции (рис. 2). Плотность воды $\rho = 1000$ кг/м^3, ускорение свободного падения g положить равным 10 м/с^2.

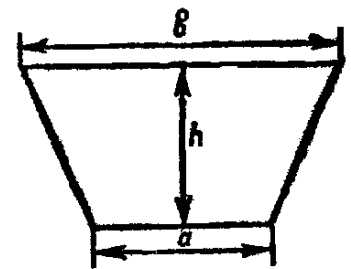У к а з а н и е. Давление на глубине x равно $\rho g x$.

Рис. 2

22.1. $a = 4{,}5$ м, $b = 6{,}6$ м, $h = 3{,}0$ м.   22.2. $a = 4{,}8$ м, $b = 7{,}2$ м, $h = 3{,}0$ м.

22.3. $a = 5{,}1$ м, $b = 7{,}8$ м, $h = 3{,}0$ м.   22.4. $a = 5{,}4$ м, $b = 8{,}4$ м, $h = 3{,}0$ м.

22.5. $a = 5{,}7$ м, $b = 9{,}0$ м, $h = 4{,}0$ м.   22.6. $a = 6{,}0$ м, $b = 9{,}6$ м, $h = 4{,}0$ м.

22.7. $a = 6{,}3$ м, $b = 10{,}2$ м, $h = 4{,}0$ м.

22.8. $a = 6{,}6$ м, $b = 10{,}8$ м, $h = 4{,}0$ м.

22.9. $a = 6{,}9$ м, $b = 11{,}4$ м, $h = 5{,}0$ м.

22.10. $a = 7{,}2$ м, $b = 12{,}0$ м, $h = 5{,}0$ м.

Варианты 11–20

Определить работу (в джоулях), совершаемую при подъеме спутника с поверхности Земли на высоту H км. Масса спутника равна m т, радиус Земли

$R_{_3} = 6380$ км. Ускорение свободного падения g у поверхности Земли положить равным 10 м/с².

22.11. $m = 7,0$ т, $H = 200$ км.
22.12. $m = 7,0$ т, $H = 250$ км.

22.13. $m = 6,0$ т, $H = 300$ км.
22.14. $m = 6,0$ т, $H = 350$ км.

22.15. $m = 5,0$ т, $H = 400$ км.
22.16. $m = 5,0$ т, $H = 450$ км.

22.17. $m = 4,0$ т, $H = 500$ км.
22.18. $m = 4,0$ т, $H = 550$ км.

22.19. $m = 3,0$ т, $H = 600$ км.
22.20. $m = 3,0$ т, $H = 650$ км.

Варианты 21–31

Цилиндр наполнен газом под атмосферным давлением (103,3 кПа). Считая газ идеальным, определить работу (в джоулях) при изотермическом сжатии газа поршнем, переместившимся внутрь цилиндра на h м (рис. 3).
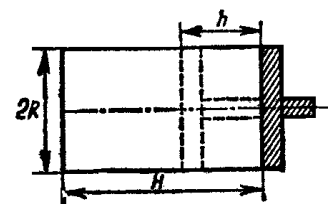
У к а з а н и е. Уравнение состояния газа $pV = \text{const}$, где p – давление, V – объем.

Рис. 3

22.21. $H = 0,4$ м, $h = 0,35$ м, $R = 0,1$ м.

22.22. $H = 0,4$ м, $h = 0,3$ м, $R = 0,1$ м.

22.23. $H = 0,4$ м, $h = 0,2$ м, $R = 0,1$ м.

22.24. $H = 0,8$ м, $h = 0,7$ м, $R = 0,2$ м.

22.25. $H = 0,8$ м, $h = 0,6$ м, $R = 0,2$ м.

22.26. $H = 0,8$ м, $h = 0,4$ м, $R = 0,2$ м.

22.27. $H = 1,6$ м, $h = 1,4$ м, $R = 0,3$ м.

22.28. $H = 1,6$ м, $h = 1,2$ м, $R = 0,3$ м.

22.29. $H = 1,6$ м, $h = 0,8$ м, $R = 0,3$ м.

22.30. $H = 2,0$ м, $h = 1,5$ м, $R = 0,4$ м.

22.31. $H = 2,0$ м, $h = 1,0$ м, $R = 0,4$ м.