

І. ПРЕДЕЛЫ

Расчетные задания

Задача 1. Доказать, что $\lim_{n \rightarrow \infty} a_n = a$ (указать $N(\varepsilon)$).

$$1.1. a_n = \frac{3n-2}{2n-1}, \quad a = \frac{3}{2}.$$

$$1.3. a_n = \frac{7n+4}{2n+1}, \quad a = \frac{7}{2}.$$

$$1.5. a_n = \frac{7n-1}{n+1}, \quad a = 7.$$

$$1.7. a_n = \frac{9-n^3}{1+2n^3}, \quad a = -\frac{1}{2}.$$

$$1.9. a_n = \frac{1-2n^2}{2+4n^2}, \quad a = -\frac{1}{2}.$$

$$1.11. a_n = \frac{n+1}{1-2n}, \quad a = -\frac{1}{2}.$$

$$1.13. a_n = \frac{1-2n^2}{n^2+3}, \quad a = -2.$$

$$1.15. a_n = \frac{n}{3n-1}, \quad a = \frac{1}{3}.$$

$$1.17. a_n = \frac{4+2n}{1-3n}, \quad a = -\frac{2}{3}.$$

$$1.19. a_n = \frac{3-n^2}{1+2n^2}, \quad a = -\frac{1}{2}.$$

$$1.21. a_n = \frac{3n-1}{5n+1}, \quad a = \frac{3}{5}.$$

$$1.23. a_n = \frac{1-2n^2}{2+4n^2}, \quad a = -\frac{1}{2}.$$

$$1.25. a_n = \frac{2-2n}{3+4n}, \quad a = -\frac{1}{2}.$$

$$1.27. a_n = \frac{1+3n}{6-n}, \quad a = -3.$$

$$1.29. a_n = \frac{3n^2+2}{4n^2-1}, \quad a = \frac{3}{4}.$$

$$1.2. a_n = \frac{4n-1}{2n+1}, \quad a = 2.$$

$$1.4. a_n = \frac{2n-5}{3n+1}, \quad a = \frac{2}{3}.$$

$$1.6. a_n = \frac{4n^2+1}{3n^2+2}, \quad a = \frac{4}{3}.$$

$$1.8. a_n = \frac{4n-3}{2n+1}, \quad a = 2.$$

$$1.10. a_n = -\frac{5n}{n+1}, \quad a = -5.$$

$$1.12. a_n = \frac{2n+1}{3n-5}, \quad a = \frac{2}{3}.$$

$$1.14. a_n = \frac{3n^2}{2-n^2}, \quad a = -3.$$

$$1.16. a_n = \frac{3n^3}{n^3-1}, \quad a = 3.$$

$$1.18. a_n = \frac{5n+15}{6-n}, \quad a = -5.$$

$$1.20. a_n = \frac{2n-1}{2-3n}, \quad a = -\frac{2}{3}.$$

$$1.22. a_n = \frac{4n-3}{2n+1}, \quad a = 2.$$

$$1.24. a_n = \frac{5n+1}{10n-3}, \quad a = \frac{1}{2}.$$

$$1.26. a_n = \frac{23-4n}{2-n}, \quad a = 4.$$

$$1.28. a_n = \frac{2n+3}{n+5}, \quad a = 2.$$

$$1.30. a_n = \frac{2-3n^2}{4+5n^2}, \quad a = -\frac{3}{5}.$$

$$1.31. a_n = \frac{2n^3}{n^3 - 2}, \quad a = 2.$$

Задача 2. Вычислить пределы числовых последовательностей.

$$2.1. \lim_{n \rightarrow \infty} \frac{(3-n)^2 + (3+n)^2}{(3-n)^2 - (3+n)^2}.$$

$$2.3. \lim_{n \rightarrow \infty} \frac{(3-n)^4 - (2-n)^4}{(1-n)^3 - (1+n)^3}.$$

$$2.5. \lim_{n \rightarrow \infty} \frac{(6-n)^2 - (6+n)^2}{(6+n)^2 - (1-n)^2}.$$

$$2.7. \lim_{n \rightarrow \infty} \frac{(1+2n)^3 - 8n^3}{(1+2n)^2 + 4n^2}.$$

$$2.9. \lim_{n \rightarrow \infty} \frac{(3-n)^3}{(n+1)^2 - (n+1)^3}.$$

$$2.11. \lim_{n \rightarrow \infty} \frac{2(n+1)^3 - (n-2)^3}{n^2 + 2n - 3}.$$

$$2.13. \lim_{n \rightarrow \infty} \frac{(n+3)^3 + (n+4)^3}{(n+3)^4 - (n+4)^4}.$$

$$2.15. \lim_{n \rightarrow \infty} \frac{8n^3 - 2n}{(n+1)^4 - (n-1)^4}.$$

$$2.17. \lim_{n \rightarrow \infty} \frac{(2n-3)^3 - (n+5)^3}{(3n-1)^3 + (2n+3)^3}.$$

$$2.19. \lim_{n \rightarrow \infty} \frac{(2n+1)^3 + (3n+2)^3}{(2n+3)^3 - (n-7)^3}.$$

$$2.21. \lim_{n \rightarrow \infty} \frac{(2n+1)^3 - (2n+3)^3}{(2n+1)^2 + (2n+3)^2}.$$

$$2.23. \lim_{n \rightarrow \infty} \frac{(n+2)^4 - (n-2)^4}{(n+5)^2 + (n-5)^2}.$$

$$2.2. \lim_{n \rightarrow \infty} \frac{(3-n)^4 - (2-n)^4}{(1-n)^4 - (1+n)^4}.$$

$$2.4. \lim_{n \rightarrow \infty} \frac{(1-n)^4 - (1+n)^4}{(1+n)^3 - (1-n)^3}.$$

$$2.6. \lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n+1)^2}{(n-1)^3 - (n+1)^3}.$$

$$2.8. \lim_{n \rightarrow \infty} \frac{(3-4n)^2}{(n-3)^3 - (n+3)^3}.$$

$$2.10. \lim_{n \rightarrow \infty} \frac{(n+1)^2 + (n-1)^2 - (n+2)^3}{(4-n)^3}.$$

$$2.12. \lim_{n \rightarrow \infty} \frac{(n+1)^3 + (n+2)^3}{(n+4)^3 + (n+5)^3}.$$

$$2.14. \lim_{n \rightarrow \infty} \frac{(n+1)^4 - (n-1)^4}{(n+1)^3 + (n-1)^3}.$$

$$2.16. \lim_{n \rightarrow \infty} \frac{(n+6)^3 - (n+1)^3}{(2n+3)^2 + (n+4)^2}.$$

$$2.18. \lim_{n \rightarrow \infty} \frac{(n+10)^2 + (3n+1)^2}{(n+6)^3 - (n+1)^3}.$$

$$2.20. \lim_{n \rightarrow \infty} \frac{(n+7)^3 - (n+2)^3}{(3n+2)^2 + (4n+1)^2}.$$

$$2.22. \lim_{n \rightarrow \infty} \frac{n^3 - (n-1)^3}{(n+1)^4 - n^4}.$$

$$2.24. \lim_{n \rightarrow \infty} \frac{(n+1)^4 - (n-1)^4}{(n+1)^3 + (n-1)^3}.$$

$$2.25. \lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n-1)^3}{(n+1)^2 - (n-1)^2}.$$

$$2.27. \lim_{n \rightarrow \infty} \frac{(n+2)^3 + (n-2)^3}{n^4 + 2n^2 - 1}.$$

$$2.29. \lim_{n \rightarrow \infty} \frac{(n+1)^3 + (n-1)^3}{n^3 + 1}.$$

$$2.31. \lim_{n \rightarrow \infty} \frac{(2n+1)^2 - (n+1)^2}{n^2 + n + 1}.$$

$$2.26. \lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n-1)^3}{(n+1)^2 + (n-1)^2}.$$

$$2.28. \lim_{n \rightarrow \infty} \frac{(n+1)^3 + (n-1)^3}{n^3 - 3n}.$$

$$2.30. \lim_{n \rightarrow \infty} \frac{(n+2)^2 - (n-2)^2}{(n+3)^2}.$$

Задача 3. Вычислить пределы числовых последовательностей.

$$3.1. \lim_{n \rightarrow \infty} \frac{n \sqrt[3]{5n^2} + \sqrt[4]{9n^8 + 1}}{(n + \sqrt{n}) \sqrt{7 - n + n^2}}.$$

$$3.3. \lim_{n \rightarrow \infty} \frac{\sqrt{n^3 + 1} - \sqrt{n-1}}{\sqrt[3]{n^3 + 1} - \sqrt{n-1}}.$$

$$3.5. \lim_{n \rightarrow \infty} \frac{\sqrt{3n-1} - \sqrt[3]{125n^3 + n}}{\sqrt[5]{n} - n}.$$

$$3.7. \lim_{n \rightarrow \infty} \frac{\sqrt{n+2} - \sqrt{n^2+2}}{\sqrt[4]{4n^4+1} - \sqrt[3]{n^4-1}}.$$

$$3.9. \lim_{n \rightarrow \infty} \frac{6n^3 - \sqrt{n^5+1}}{\sqrt{4n^6+3} - n}.$$

$$3.11. \lim_{n \rightarrow \infty} \frac{n \sqrt[4]{3n+1} + \sqrt{81n^4 - n^2 + 1}}{(n + \sqrt[3]{n}) \sqrt{5 - n + n^2}}.$$

$$3.13. \lim_{n \rightarrow \infty} \frac{\sqrt{n^5+3} - \sqrt{n-3}}{\sqrt[5]{n^5+3} + \sqrt{n-3}}.$$

$$3.15. \lim_{n \rightarrow \infty} \frac{\sqrt{4n+1} - \sqrt[3]{27n^3+4}}{\sqrt[4]{n} - \sqrt[3]{n^5+n}}.$$

$$3.2. \lim_{n \rightarrow \infty} \frac{\sqrt{n-1} - \sqrt{n^2+1}}{\sqrt[3]{3n^3+3} + \sqrt[4]{n^5+1}}.$$

$$3.4. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2-1} + 7n^3}{\sqrt[4]{n^{12}+n+1} - n}.$$

$$3.6. \lim_{n \rightarrow \infty} \frac{n \sqrt[5]{n} - \sqrt[3]{27n^6+n^2}}{(n + \sqrt[4]{n}) \sqrt{9+n^2}}.$$

$$3.8. \lim_{n \rightarrow \infty} \frac{\sqrt{n^4+2} + \sqrt{n-2}}{\sqrt[4]{n^4+2} + \sqrt{n-2}}.$$

$$3.10. \lim_{n \rightarrow \infty} \frac{\sqrt{5n+2} - \sqrt[3]{8n^3+5}}{\sqrt[4]{n+7} - n}.$$

$$3.12. \lim_{n \rightarrow \infty} \frac{\sqrt{n+3} - \sqrt{n^2-3}}{\sqrt[3]{n^5-4} - \sqrt[4]{n^4+1}}.$$

$$3.14. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n} - 9n^2}{3n - \sqrt[4]{9n^8+1}}.$$

$$3.16. \lim_{n \rightarrow \infty} \frac{n \sqrt[3]{7n} - \sqrt[4]{81n^8-1}}{(n + 4\sqrt{n}) \sqrt{n^2-5}}.$$

$$3.17. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^3 - 7} + \sqrt[3]{n^2 + 4}}{\sqrt[4]{n^5 + 5} + \sqrt{n}}$$

$$3.18. \lim_{n \rightarrow \infty} \frac{\sqrt{n^6 + 4} + \sqrt{n - 4}}{\sqrt[5]{n^6 + 6} - \sqrt{n - 6}}$$

$$3.19. \lim_{n \rightarrow \infty} \frac{4n^2 - \sqrt[4]{n^3}}{\sqrt[3]{n^6 + n^3 + 1} - 5n}$$

$$3.20. \lim_{n \rightarrow \infty} \frac{\sqrt{n + 3} - \sqrt[3]{8n^3 + 3}}{\sqrt[4]{n + 4} - \sqrt[5]{n^5 + 5}}$$

$$3.21. \lim_{n \rightarrow \infty} \frac{n \sqrt[4]{11n} + \sqrt{25n^4 - 81}}{(n - 7\sqrt{n})\sqrt{n^2 - n + 1}}$$

$$3.22. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2} - \sqrt{n^2 + 5}}{\sqrt[5]{n^7} - \sqrt{n + 1}}$$

$$3.23. \lim_{n \rightarrow \infty} \frac{\sqrt{n^7 + 5} - \sqrt{n - 5}}{\sqrt[7]{n^7 + 5} + \sqrt{n - 5}}$$

$$3.24. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 + 2} - 5n^2}{n - \sqrt{n^4 - n + 1}}$$

$$3.25. \lim_{n \rightarrow \infty} \frac{\sqrt{n + 2} - \sqrt[3]{n^3 + 2}}{\sqrt[7]{n + 2} - \sqrt[5]{n^5 + 2}}$$

$$3.26. \lim_{n \rightarrow \infty} \frac{n \sqrt{71n} - \sqrt[3]{64n^6 + 9}}{(n - \sqrt[3]{n})\sqrt{11 + n^2}}$$

$$3.27. \lim_{n \rightarrow \infty} \frac{\sqrt{n + 6} - \sqrt{n^2 - 5}}{\sqrt[3]{n^3 + 3} + \sqrt[4]{n^3 + 1}}$$

$$3.28. \lim_{n \rightarrow \infty} \frac{\sqrt{n^8 + 6} - \sqrt{n - 6}}{\sqrt[8]{n^8 + 6} + \sqrt{n - 6}}$$

$$3.29. \lim_{n \rightarrow \infty} \frac{n^2 - \sqrt{n^3 + 1}}{\sqrt[3]{n^6 + 2} - n}$$

$$3.30. \lim_{n \rightarrow \infty} \frac{\sqrt{n + 1} - \sqrt[3]{n^3 + 1}}{\sqrt[4]{n + 1} - \sqrt[5]{n^5 + 1}}$$

$$3.31. \lim_{n \rightarrow \infty} \frac{n \sqrt[6]{n} + \sqrt[3]{n^{10} + 1}}{(n + \sqrt[4]{n}) \sqrt[3]{n^3 - 1}}$$

Задача 4. Вычислить пределы числовых последовательностей.

$$4.1. \lim_{n \rightarrow \infty} n(\sqrt{n^2 + 1} + \sqrt{n^2 - 1}).$$

$$4.2. \lim_{n \rightarrow \infty} n(\sqrt{n(n - 2)} - \sqrt{n^2 - 3}).$$

$$4.3. \lim_{n \rightarrow \infty} (n - \sqrt[3]{n^3 - 5})n\sqrt{n}.$$

$$4.4. \lim_{n \rightarrow \infty} \left[\sqrt{(n^2 + 1)(n^2 - 4)} - \sqrt{n^4 - 9} \right]$$

$$4.5. \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 - 8} - n\sqrt{n(n^2 + 5)}}{\sqrt{n}}$$

$$4.6. \lim_{n \rightarrow \infty} (\sqrt{n^2 - 3n + 2} - n).$$

$$4.7. \lim_{n \rightarrow \infty} (n + \sqrt[3]{4 - n^3}).$$

$$4.8. \lim_{n \rightarrow \infty} \left[\sqrt{n(n + 2)} - \sqrt{n^2 - 2n + 3} \right].$$

$$4.9. \lim_{n \rightarrow \infty} \left[\sqrt{(n + 2)(n + 1)} - \sqrt{(n - 1)(n + 3)} \right].$$

$$4.10. \lim_{n \rightarrow \infty} n^2 \left(\sqrt{n(n^4 - 1)} - \sqrt{n^5 - 8} \right).$$

$$4.11. \lim_{n \rightarrow \infty} n \left(\sqrt[3]{5 + 8n^3} - 2n \right).$$

$$4.13. \lim_{n \rightarrow \infty} \left[\sqrt[3]{(n+2)^2} - \sqrt[3]{(n-3)^2} \right].$$

$$4.15. \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 3n - 2} - \sqrt{n^2 - 3} \right).$$

$$4.17. \lim_{n \rightarrow \infty} \frac{\sqrt{n(n^5 + 9)} - \sqrt{(n^4 - 1)(n^2 + 5)}}{n}.$$

$$4.18. \lim_{n \rightarrow \infty} \left(\sqrt{n(n+5)} - n \right).$$

$$4.20. \lim_{n \rightarrow \infty} \frac{\sqrt{(n^3 + 1)(n^2 + 3)} - \sqrt{n(n^4 + 2)}}{2\sqrt{n}}.$$

$$4.21. \lim_{n \rightarrow \infty} \left[\sqrt{(n^2 + 1)(n^2 + 2)} - \sqrt{(n^2 - 1)(n^2 - 2)} \right].$$

$$4.22. \lim_{n \rightarrow \infty} \frac{\sqrt{(n^5 + 1)(n^2 - 1)} - n\sqrt{n(n^4 + 1)}}{n}.$$

$$4.23. \lim_{n \rightarrow \infty} \frac{\sqrt{(n^4 + 1)(n^2 - 1)} - \sqrt{n^6 - 1}}{n}.$$

$$4.25. \lim_{n \rightarrow \infty} n^3 \left(\sqrt[3]{n^2(n^6 + 4)} - \sqrt[3]{(n^8 - 1)} \right).$$

$$4.27. \lim_{n \rightarrow \infty} \sqrt[3]{n} \left(\sqrt[3]{n^2} - \sqrt[3]{n(n-1)} \right).$$

$$4.29. \lim_{n \rightarrow \infty} n \left(\sqrt{n^4 + 3} - \sqrt{n^4 - 2} \right).$$

$$4.30. \lim_{n \rightarrow \infty} \sqrt{n(n+1)(n+2)} \left(\sqrt{n^3 - 3} - \sqrt{n^3 - 2} \right).$$

$$4.31. \lim_{n \rightarrow \infty} \frac{\sqrt{(n^2 + 5)(n^4 + 2)} - \sqrt{n^6 - 3n^3 + 5}}{n}.$$

$$4.12. \lim_{n \rightarrow \infty} n^2 \left(\sqrt[3]{5 + n^3} - \sqrt[3]{3 + n^3} \right).$$

$$4.14. \lim_{n \rightarrow \infty} \frac{\sqrt{(n+1)^3} - \sqrt{n(n-1)(n-3)}}{\sqrt{n}}.$$

$$4.16. \lim_{n \rightarrow \infty} \sqrt{n} \left(\sqrt{n+2} - \sqrt{n-3} \right).$$

$$4.19. \lim_{n \rightarrow \infty} \sqrt{n^3 + 8} \left(\sqrt{n^3 + 2} - \sqrt{n^3 - 1} \right).$$

$$4.24. \lim_{n \rightarrow \infty} \left[n - \sqrt{n(n-1)} \right].$$

$$4.26. \lim_{n \rightarrow \infty} \left[n\sqrt{n} - \sqrt{n(n+1)(n+2)} \right].$$

$$4.28. \lim_{n \rightarrow \infty} \sqrt{n+2} \left(\sqrt{n+3} - \sqrt{n-4} \right).$$

Задача 5. Вычислить пределы числовых последовательностей.

$$5.1. \lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \dots + \frac{n-1}{n^2} \right).$$

$$5.2. \lim_{n \rightarrow \infty} \frac{(2n+1)! + (2n+2)!}{(2n+3)!}.$$

$$5.3. \lim_{n \rightarrow \infty} \left[\frac{1+3+5+7+\dots+(2n-1)}{n+1} - \frac{2n+1}{2} \right].$$

$$5.4. \lim_{n \rightarrow \infty} \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n}.$$

$$5.5. \lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}.$$

$$5.6. \lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}.$$

$$5.7. \lim_{n \rightarrow \infty} \left[\frac{1+3+5+7+\dots+(2n-1)}{n+3} - n \right].$$

$$5.8. \lim_{n \rightarrow \infty} \frac{1+4+7+\dots+(3n-2)}{\sqrt{5n^4+n+1}}.$$

$$5.9. \lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!}.$$

$$5.10. \lim_{n \rightarrow \infty} \frac{(3n-1)! + (3n+1)!}{(3n)!(n-1)}.$$

$$5.11. \lim_{n \rightarrow \infty} \frac{2^n - 5^{n+1}}{2^{n+1} + 5^{n+2}}.$$

$$5.12. \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{3} + \frac{1}{3^2} + \dots + \frac{1}{3^n}}{1 + \frac{1}{5} + \frac{1}{5^2} + \dots + \frac{1}{5^n}}.$$

$$5.13. \lim_{n \rightarrow \infty} \frac{1-3+5-7+9-11+\dots+(4n-3)-(4n-1)}{\sqrt{n^2+1} + \sqrt{n^2+n+1}}.$$

$$5.14. \lim_{n \rightarrow \infty} \frac{1-2+3-4+\dots+(2n-1)-2n}{\sqrt{9n^4+1}}.$$

$$5.15. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^3+5} - \sqrt{3n^4+2}}{1+3+5+\dots+(2n-1)}.$$

$$5.16. \lim_{n \rightarrow \infty} \frac{3^n - 2^n}{3^{n-1} + 2^n}.$$

$$5.17. \lim_{n \rightarrow \infty} \left[\frac{n+2}{1+2+3+\dots+n} - \frac{2}{3} \right].$$

$$5.18. \lim_{n \rightarrow \infty} \left(\frac{5}{6} + \frac{13}{36} + \dots + \frac{3^n + 2^n}{6^n} \right).$$

$$5.19. \lim_{n \rightarrow \infty} \frac{2 - 5 + 4 - 7 + \dots + 2n - (2n + 3)}{n + 3}.$$

$$5.20. \lim_{n \rightarrow \infty} \frac{(2n + 1)! + (2n + 2)!}{(2n + 3)! - (2n + 2)!}.$$

$$5.21. \lim_{n \rightarrow \infty} \frac{1 + 2 + \dots + n}{n - n^2 + 3}.$$

$$5.22. \lim_{n \rightarrow \infty} \frac{n^2 + \sqrt{n} - 1}{2 + 7 + 12 + \dots + (5n - 3)}.$$

$$5.23. \lim_{n \rightarrow \infty} \left(\frac{3}{4} + \frac{5}{16} + \frac{9}{64} + \dots + \frac{1 + 2^n}{4^n} \right).$$

$$5.24. \lim_{n \rightarrow \infty} \frac{2 + 4 + 6 + \dots + 2n}{1 + 3 + 5 + \dots + (2n - 1)}.$$

$$5.25. \lim_{n \rightarrow \infty} \left[\frac{1 + 5 + 9 + 13 + \dots + (4n - 3)}{n + 1} - \frac{4n + 1}{2} \right].$$

$$5.26. \lim_{n \rightarrow \infty} \frac{1 - 2 + 3 - 4 + \dots - 2n}{\sqrt[3]{n^3 + 2n + 2}}.$$

$$5.27. \lim_{n \rightarrow \infty} \frac{2^n + 7^n}{2^n - 7^{n-1}}.$$

$$5.28. \lim_{n \rightarrow \infty} \frac{n! + (n + 2)!}{(n - 1)! + (n + 2)!}.$$

$$5.29. \lim_{n \rightarrow \infty} \frac{3 + 6 + 9 + \dots + 3n}{n^2 + 4}.$$

$$5.30. \lim_{n \rightarrow \infty} \left(\frac{7}{10} + \frac{29}{100} + \dots + \frac{2^n + 5^n}{10^n} \right).$$

$$5.31. \lim_{n \rightarrow \infty} \left(\frac{2 + 4 + \dots + 2n}{n + 3} - n \right).$$

Задача 6. Вычислить пределы числовых последовательностей.

$$6.1. \lim_{n \rightarrow \infty} \left(\frac{n + 1}{n - 1} \right)^n.$$

$$6.2. \lim_{n \rightarrow \infty} \left(\frac{2n + 3}{2n + 1} \right)^{n+1}.$$

$$6.3. \lim_{n \rightarrow \infty} \left(\frac{n^2 - 1}{n^2} \right)^{n^4}.$$

$$6.4. \lim_{n \rightarrow \infty} \left(\frac{n - 1}{n + 3} \right)^{n+2}.$$

$$6.5. \lim_{n \rightarrow \infty} \left(\frac{2n^2 + 2}{2n^2 + 1} \right)^{n^2}.$$

$$6.6. \lim_{n \rightarrow \infty} \left(\frac{3n^2 - 6n + 7}{3n^2 + 20n - 1} \right)^{-n+1}.$$

$$6.7. \lim_{n \rightarrow \infty} \left(\frac{n^2 - 3n + 6}{n^2 + 5n + 1} \right)^{n/2}.$$

$$6.8. \lim_{n \rightarrow \infty} \left(\frac{n - 10}{n + 1} \right)^{3n+1}.$$

$$6.9. \lim_{n \rightarrow \infty} \left(\frac{6n - 7}{6n + 4} \right)^{3n+2}.$$

$$6.10. \lim_{n \rightarrow \infty} \left(\frac{3n^2 + 4n - 1}{3n^2 + 2n + 7} \right)^{2n+5}.$$

$$6.11. \lim_{n \rightarrow \infty} \left(\frac{n^2 + n + 1}{n^2 + n - 1} \right)^{-n^2}.$$

$$6.12. \lim_{n \rightarrow \infty} \left(\frac{2n^2 + 5n + 7}{2n^2 + 5n + 3} \right)^n.$$

$$6.13. \lim_{n \rightarrow \infty} \left(\frac{n-1}{n+1} \right)^{n^2}.$$

$$6.14. \lim_{n \rightarrow \infty} \left(\frac{5n^2 + 3n - 1}{5n^2 + 3n + 3} \right)^{n^2}.$$

$$6.15. \lim_{n \rightarrow \infty} \left(\frac{3n+1}{3n-1} \right)^{2n+3}.$$

$$6.16. \lim_{n \rightarrow \infty} \left(\frac{2n^2 + 7n - 1}{2n^2 + 3n - 1} \right)^{-n^2}.$$

$$6.17. \lim_{n \rightarrow \infty} \left(\frac{n+3}{n+5} \right)^{n+4}.$$

$$6.18. \lim_{n \rightarrow \infty} \left(\frac{n^3 + 1}{n^3 - 1} \right)^{2n-n^3}.$$

$$6.19. \lim_{n \rightarrow \infty} \left(\frac{2n^2 + 21n - 7}{2n^2 + 18n + 9} \right)^{2n+1}.$$

$$6.20. \lim_{n \rightarrow \infty} \left(\frac{10n-3}{10n-1} \right)^{5n}.$$

$$6.21. \lim_{n \rightarrow \infty} \left(\frac{3n^2 - 5n}{3n^2 - 5n + 7} \right)^{n+1}.$$

$$6.22. \lim_{n \rightarrow \infty} \left(\frac{n+3}{n+1} \right)^{-n^2}.$$

$$6.23. \lim_{n \rightarrow \infty} \left(\frac{n^2 - 6n + 5}{n^2 - 5n + 5} \right)^{3n+2}.$$

$$6.24. \lim_{n \rightarrow \infty} \left(\frac{n+4}{n+2} \right)^n.$$

$$6.25. \lim_{n \rightarrow \infty} \left(\frac{7n^2 + 18n - 15}{7n^2 + 11n + 15} \right)^{n+2}.$$

$$6.26. \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+1} \right)^{n+1}.$$

$$6.27. \lim_{n \rightarrow \infty} \left(\frac{n^3 + n + 1}{n^3 + 2} \right)^{2n^2}.$$

$$6.28. \lim_{n \rightarrow \infty} \left(\frac{13n+3}{13n-10} \right)^{n-3}.$$

$$6.29. \lim_{n \rightarrow \infty} \left(\frac{2n^2 + 2n + 3}{2n^2 + 2n + 1} \right)^{3n^2-7}.$$

$$6.30. \lim_{n \rightarrow \infty} \left(\frac{n+5}{n-7} \right)^{n/6+1}.$$

$$6.31. \lim_{n \rightarrow \infty} \left(\frac{4n^2 + 4n - 1}{4n^2 + 2n + 3} \right)^{1-2n}.$$

Задача 7. Доказать (найти $\delta(\varepsilon)$), что:

$$7.1. \lim_{x \rightarrow -3} \frac{2x^2 + 5x - 3}{x + 3} = -7.$$

$$7.2. \lim_{x \rightarrow 1} \frac{5x^2 - 4x - 1}{x - 1} = 6.$$

$$7.3. \lim_{x \rightarrow -2} \frac{3x^2 + 5x - 2}{x + 2} = -7.$$

$$7.5. \lim_{x \rightarrow -1/2} \frac{6x^2 + x - 1}{x + 1/2} = -5.$$

$$7.7. \lim_{x \rightarrow -1/3} \frac{9x^2 - 1}{x + 1/3} = -6.$$

$$7.9. \lim_{x \rightarrow -1/3} \frac{3x^2 - 2x - 1}{x + 1/3} = -4.$$

$$7.11. \lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x - 3} = 2.$$

$$7.13. \lim_{x \rightarrow 1/3} \frac{6x^2 - 5x + 1}{x - 1/3} = -1.$$

$$7.15. \lim_{x \rightarrow -7/2} \frac{2x^2 + 13x + 21}{2x + 7} = -\frac{1}{2}.$$

$$7.17. \lim_{x \rightarrow 1/3} \frac{6x^2 + x - 1}{x - 1/3} = 5.$$

$$7.19. \lim_{x \rightarrow 11} \frac{2x^2 - 21x - 11}{x - 11} = 23.$$

$$7.21. \lim_{x \rightarrow -7} \frac{2x^2 + 15x + 7}{x + 7} = -13.$$

$$7.23. \lim_{x \rightarrow -1/3} \frac{6x^2 - x - 1}{3x + 1} = -\frac{5}{3}.$$

$$7.25. \lim_{x \rightarrow 8} \frac{3x^2 - 40x + 128}{x - 8} = 8.$$

$$7.27. \lim_{x \rightarrow 1/2} \frac{2x^2 - 5x + 2}{x - 1/2} = -3.$$

$$7.29. \lim_{x \rightarrow 1/3} \frac{3x^2 + 17x - 6}{x - 1/3} = 19.$$

$$7.4. \lim_{x \rightarrow 3} \frac{4x^2 - 14x + 6}{x - 3} = 10.$$

$$7.6. \lim_{x \rightarrow 1/2} \frac{6x^2 - x - 1}{x - 1/2} = 5.$$

$$7.8. \lim_{x \rightarrow 2} \frac{3x^2 - 5x - 2}{x - 2} = 7.$$

$$7.10. \lim_{x \rightarrow -1} \frac{7x^2 + 8x + 1}{x + 1} = -6.$$

$$7.12. \lim_{x \rightarrow 1/2} \frac{2x^2 + 3x - 2}{x - 1/2} = 5.$$

$$7.14. \lim_{x \rightarrow -7/5} \frac{10x^2 + 9x - 7}{x + 7/5} = -19.$$

$$7.16. \lim_{x \rightarrow 5/2} \frac{2x^2 - 9x + 10}{2x - 5} = \frac{1}{2}.$$

$$7.18. \lim_{x \rightarrow -1/2} \frac{6x^2 - 75x - 39}{x + 1/2} = -81.$$

$$7.20. \lim_{x \rightarrow 5} \frac{5x^2 - 24x - 5}{x - 5} = 26.$$

$$7.22. \lim_{x \rightarrow -4} \frac{2x^2 + 6x - 8}{x + 4} = -10.$$

$$7.24. \lim_{x \rightarrow -5} \frac{x^2 + 2x - 15}{x + 5} = -8.$$

$$7.26. \lim_{x \rightarrow 10} \frac{5x^2 - 51x + 10}{x - 10} = 49.$$

$$7.28. \lim_{x \rightarrow -6} \frac{3x^2 + 17x - 6}{x + 6} = -19.$$

$$7.30. \lim_{x \rightarrow -1/5} \frac{15x^2 - 2x - 1}{x + 1/5} = -8.$$

$$7.31. \lim_{x \rightarrow 1/3} \frac{15x^2 - 2x - 1}{x - 1/3} = 8.$$

Задача 8. Доказать, что функция $f(x)$ непрерывна в точке x_0 (найти $\delta(\varepsilon)$).

$$8.1. f(x) = 5x^2 - 1, x_0 = 6.$$

$$8.2. f(x) = 4x^2 - 2, x_0 = 5.$$

$$8.3. f(x) = 3x^2 - 3, x_0 = 4.$$

$$8.4. f(x) = 2x^2 - 4, x_0 = 3.$$

$$8.5. f(x) = -2x^2 - 5, x_0 = 2.$$

$$8.6. f(x) = -3x^2 - 6, x_0 = 1.$$

$$8.7. f(x) = -4x^2 - 7, x_0 = 1.$$

$$8.8. f(x) = -5x^2 - 8, x_0 = 2.$$

$$8.9. f(x) = -5x^2 - 9, x_0 = 3.$$

$$8.10. f(x) = -4x^2 + 9, x_0 = 4.$$

$$8.11. f(x) = -3x^2 + 8, x_0 = 5.$$

$$8.12. f(x) = -2x^2 + 7, x_0 = 6.$$

$$8.13. f(x) = 2x^2 + 6, x_0 = 7.$$

$$8.14. f(x) = 3x^2 + 5, x_0 = 8.$$

$$8.15. f(x) = 4x^2 + 4, x_0 = 9.$$

$$8.16. f(x) = 5x^2 + 3, x_0 = 8.$$

$$8.17. f(x) = 5x^2 + 1, x_0 = 7.$$

$$8.18. f(x) = 4x^2 - 1, x_0 = 6.$$

$$8.19. f(x) = 3x^2 - 2, x_0 = 5.$$

$$8.20. f(x) = 2x^2 - 3, x_0 = 4.$$

$$8.21. f(x) = -2x^2 - 4, x_0 = 3.$$

$$8.22. f(x) = -3x^2 - 5, x_0 = 2.$$

$$8.23. f(x) = -4x^2 - 6, x_0 = 1.$$

$$8.24. f(x) = -5x^2 - 7, x_0 = 1.$$

$$8.25. f(x) = -4x^2 - 8, x_0 = 2.$$

$$8.26. f(x) = -3x^2 - 9, x_0 = 3.$$

$$8.27. f(x) = -2x^2 + 9, x_0 = 4.$$

$$8.28. f(x) = 2x^2 + 8, x_0 = 5.$$

$$8.29. f(x) = 3x^2 + 7, x_0 = 6.$$

$$8.30. f(x) = 4x^2 + 6, x_0 = 7.$$

$$8.31. f(x) = 5x^2 + 5, x_0 = 8.$$

Задача 9. Вычислить пределы функций.

$$9.1. \lim_{x \rightarrow -1} \frac{(x^3 - 2x - 1)(x + 1)}{x^4 + 4x^2 - 5}.$$

$$9.2. \lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{x + x^2}.$$

$$9.3. \lim_{x \rightarrow -1} \frac{(x^2 + 3x + 2)^2}{x^3 + 2x^2 - x - 2}.$$

$$9.5. \lim_{x \rightarrow -3} \frac{(x^2 + 2x - 3)^2}{x^3 + 4x^2 + 3x}.$$

$$9.7. \lim_{x \rightarrow 0} \frac{(1+x)^3 - (1+3x)}{x+x^5}.$$

$$9.9. \lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{x^2 - x - 2}.$$

$$9.11. \lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^3 - x^2 - x + 1}.$$

$$9.13. \lim_{x \rightarrow -1} \frac{x^3 + 4x^2 + 5x + 2}{x^3 - 3x - 2}.$$

$$9.15. \lim_{x \rightarrow -2} \frac{x^3 + 5x^2 + 8x + 4}{x^3 + 3x^2 - 4}.$$

$$9.17. \lim_{x \rightarrow -2} \frac{x^3 - 6x^2 + 12x - 8}{x^3 - 3x^2 + 4}.$$

$$9.19. \lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{(x^2 - x - 2)^2}.$$

$$9.21. \lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{x^2 + 2x + 1}.$$

$$9.23. \lim_{x \rightarrow 1} \frac{x^4 - 1}{2x^4 - x^2 - 1}.$$

$$9.25. \lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{x^3 + 2x^2 - x - 2}.$$

$$9.27. \lim_{x \rightarrow -1} \frac{x^3 - 2x - 1}{x^4 + 2x + 1}.$$

$$9.29. \lim_{x \rightarrow 1} \frac{x^2 - 1}{2x^2 - x - 1}.$$

$$9.4. \lim_{x \rightarrow 1} \frac{(2x^2 - x - 1)^2}{x^3 + 2x^2 - x - 2}.$$

$$9.6. \lim_{x \rightarrow -1} \frac{(x^3 - 2x - 1)^2}{x^4 + 2x + 1}.$$

$$9.8. \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{2x^2 - x - 1}.$$

$$9.10. \lim_{x \rightarrow -1} \frac{x^3 + 5x^2 + 7x + 3}{x^3 + 4x^2 + 5x + 2}.$$

$$9.12. \lim_{x \rightarrow 1} \frac{x^3 + x^2 - 5x + 3}{x^3 - x^2 - x + 1}.$$

$$9.14. \lim_{x \rightarrow 1} \frac{x^4 - 1}{2x^4 - x^2 - 1}.$$

$$9.16. \lim_{x \rightarrow 2} \frac{x^3 - 5x^2 + 8x - 4}{x^3 - 3x^2 + 4}.$$

$$9.18. \lim_{x \rightarrow -2} \frac{x^3 + 5x^2 + 8x + 4}{x^3 + 7x^2 + 16x + 12}.$$

$$9.20. \lim_{x \rightarrow -2} \frac{x^3 - 3x - 2}{x - 2}.$$

$$9.22. \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 - x^2 - x + 1}.$$

$$9.24. \lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^3 + 2x^2 - x - 2}.$$

$$9.26. \lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x^3 + 4x^2 + 3x}.$$

$$9.28. \lim_{x \rightarrow 0} \frac{(1+x)^3 - (1+3x)}{x^2 + x^5}.$$

$$9.30. \lim_{x \rightarrow -3} \frac{x^3 + 7x^2 + 15x + 9}{x^3 + 8x^2 + 21x + 18}.$$

$$9.31. \lim_{x \rightarrow 3} \frac{x^3 - 4x^2 - 3x + 18}{x^3 - 5x^2 + 3x + 9}.$$

Задача 10. Вычислить пределы функций.

$$10.1 \lim_{x \rightarrow 4} \frac{\sqrt{1+2x} - 3}{\sqrt{x} - 2}.$$

$$10.2. \lim_{x \rightarrow -8} \frac{\sqrt{1-x} - 3}{2 + \sqrt[3]{x}}.$$

$$10.3 \lim_{x \rightarrow 1} \frac{\sqrt{x-1}}{\sqrt[3]{x^2-1}}.$$

$$10.4 \lim_{x \rightarrow 3} \frac{\sqrt{x+13} - 2\sqrt{x+1}}{x^2 - 9}.$$

$$10.5 \lim_{x \rightarrow -2} \frac{\sqrt[3]{x-6} + 2}{x^3 + 8}.$$

$$10.6 \lim_{x \rightarrow 16} \frac{\sqrt[4]{x} - 2}{\sqrt{x} - 4}.$$

$$10.7 \lim_{x \rightarrow 8} \frac{\sqrt{9+2x} - 5}{\sqrt[3]{x} - 2}.$$

$$10.8 \lim_{x \rightarrow 0} \frac{\sqrt{1-2x+x^2} - (1+x)}{x}.$$

$$10.9 \lim_{x \rightarrow 0} \frac{\sqrt[3]{8+3x+x^2} - 2}{x+x^2}.$$

$$10.10 \lim_{x \rightarrow 0} \frac{\sqrt[3]{27+x} - \sqrt[3]{27-x}}{x + 2\sqrt[3]{x^4}}.$$

$$10.11 \lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt{1+x} - \sqrt{2x}}.$$

$$10.12 \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt[3]{1+x} - \sqrt[3]{1-x}}.$$

$$10.13 \lim_{x \rightarrow 2} \frac{\sqrt[3]{4x} - 2}{\sqrt{2+x} - \sqrt{2x}}.$$

$$10.14 \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x^2 - 1}.$$

$$10.15 \lim_{x \rightarrow 3} \frac{\sqrt[3]{9x} - 3}{\sqrt{3+x} - \sqrt{2x}}.$$

$$10.16 \lim_{x \rightarrow -2} \frac{\sqrt[3]{x-6} + 2}{x+2}.$$

$$10.17 \lim_{x \rightarrow 4} \frac{\sqrt[3]{16x} - 4}{\sqrt{4+x} - \sqrt{2x}}.$$

$$10.18 \lim_{x \rightarrow 8} \frac{\sqrt{9+2x} - 5}{\sqrt[3]{x^2} - 4}.$$

$$10.19 \lim_{x \rightarrow 1/2} \frac{\sqrt[3]{x/4} - 1/2}{\sqrt{1/2+x} - \sqrt{2x}}.$$

$$10.20 \lim_{x \rightarrow 1/3} \frac{\sqrt[3]{x/9} - 1/3}{\sqrt{1/3+x} - \sqrt{2x}}.$$

$$10.21 \lim_{x \rightarrow 1/4} \frac{\sqrt[3]{x/16} - 1/4}{\sqrt{1/4+x} - \sqrt{2x}}.$$

$$10.22 \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt[7]{x}}.$$

$$10.23 \lim_{x \rightarrow 0} \frac{\sqrt[3]{27+x} - \sqrt[3]{27-x}}{\sqrt[3]{x^2} + \sqrt[5]{x}}.$$

$$10.24 \lim_{x \rightarrow 0} \frac{\sqrt[3]{8+3x-x^2} - 2}{\sqrt[3]{x^2} + x^3}.$$

$$10.25 \lim_{x \rightarrow 0} \frac{\sqrt{1-2x+3x^2} - (1+x)}{\sqrt[3]{x}}.$$

$$10.26 \lim_{x \rightarrow 8} \frac{\sqrt{9+2x} - 5}{\sqrt[3]{x} - 2}.$$

$$10.27 \lim_{x \rightarrow 16} \frac{\sqrt[4]{x} - 2}{\sqrt[3]{(\sqrt{x} - 4)^2}}.$$

$$10.28 \lim_{x \rightarrow -2} \frac{\sqrt[3]{x-6} + 2}{\sqrt[3]{x^3} + 8}.$$

$$10.29 \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{\sqrt[3]{x^2} - 16}.$$

$$10.30 \lim_{x \rightarrow -8} \frac{10 - x - 6\sqrt{1-x}}{2 + \sqrt[3]{x}}.$$

$$10.31 \lim_{x \rightarrow 3} \frac{\sqrt{x+13} - 2\sqrt{x+1}}{\sqrt[3]{x^2} - 9}.$$

Задача 11. Вычислить пределы функций.

$$11.1. \lim_{x \rightarrow 0} \frac{\ln(1 + \sin x)}{\sin 4x}.$$

$$11.2. \lim_{x \rightarrow 0} \frac{1 - \cos 10x}{e^{x^2} - 1}.$$

$$11.3 \lim_{x \rightarrow 0} \frac{3x^2 - 5x}{\sin 3x}.$$

$$11.4 \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{\cos 7x - \cos 3x}.$$

$$11.5 \lim_{x \rightarrow 0} \frac{4x}{\operatorname{tg}(\pi(2+x))}.$$

$$11.6 \lim_{x \rightarrow 0} \frac{2x}{\operatorname{tg}[2\pi(x+1/2)]}.$$

$$11.7 \lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{4x^2}.$$

$$11.8 \lim_{x \rightarrow 0} \frac{\arcsin 3x}{\sqrt{2+x} - \sqrt{2}}.$$

$$11.9 \lim_{x \rightarrow 0} \frac{2^x - 1}{\ln(1+2x)}.$$

$$11.10 \lim_{x \rightarrow 0} \frac{\operatorname{arctg} 2x}{\sin(2\pi(x+10))}.$$

$$11.11 \lim_{x \rightarrow 0} \frac{\ln(1-7x)}{\sin(\pi(x+7))}.$$

$$11.12 \lim_{x \rightarrow 0} \frac{\cos(x+5\pi/2)\operatorname{tg} x}{\arcsin 2x^2}.$$

$$11.13 \lim_{x \rightarrow 0} \frac{9\ln(1-2x)}{4\operatorname{arctg} 3x}.$$

$$11.14 \lim_{x \rightarrow 0} \frac{1 - \sqrt{3x+1}}{\cos[\pi(x+1)/2]}.$$

$$11.15 \lim_{x \rightarrow 0} \frac{\sin 7x}{x^2 + \pi x}.$$

$$11.16 \lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{3\operatorname{arctg} x}.$$

$$11.17 \lim_{x \rightarrow 0} \frac{2 \sin[\pi(x+1)]}{\ln(1+2x)}.$$

$$11.18 \lim_{x \rightarrow 0} \frac{\cos 2x - \cos x}{1 - \cos x}.$$

$$11.19 \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{\sin[\pi(x+2)]}.$$

$$11.20 \lim_{x \rightarrow 0} \frac{\sin[5(x+\pi)]}{e^{3x} - 1}.$$

$$11.21 \lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{x \sin x}.$$

$$11.22 \lim_{x \rightarrow 0} \frac{\arcsin 2x}{2^{-3x} - 1} \ln 2.$$

$$11.23 \lim_{x \rightarrow 0} \frac{e^{4x} - 1}{\sin(\pi(x/2 + 1))}.$$

$$11.24 \lim_{x \rightarrow 0} \frac{1 - \cos x}{(e^{3x} - 1)^2}.$$

$$11.25 \lim_{x \rightarrow 0} \frac{\sin^2 x - \operatorname{tg}^2 x}{x^4}.$$

$$11.26 \lim_{x \rightarrow 0} \frac{\arcsin 2x}{\ln(e-x) - 1}.$$

$$11.27 \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{x(1 - \cos 2x)}.$$

$$11.28 \lim_{x \rightarrow 0} \frac{\ln(x^2 + 1)}{1 - \sqrt{x^2 + 1}}.$$

$$11.29 \lim_{x \rightarrow 0} \frac{\operatorname{tg}(\pi(1+x/2))}{\ln(x+1)}.$$

$$11.30 \lim_{x \rightarrow 0} \frac{2(e^{\pi x} - 1)}{3(\sqrt[3]{1+x} - 1)}.$$

$$11.31 \lim_{x \rightarrow 0} \frac{2x \sin x}{1 - \cos x}.$$

Задача 12. Вычислить пределы функций.

$$12.1. \lim_{x \rightarrow 1} \frac{x^2 - 1}{\ln x}.$$

$$12.2. \lim_{x \rightarrow 1} \frac{\sqrt{x^2 - x + 1} - 1}{\ln x}.$$

$$12.3 \lim_{x \rightarrow \pi} \frac{1 + \cos 3x}{\sin^2 7x}.$$

$$12.4 \lim_{x \rightarrow \pi/4} \frac{1 - \sin 2x}{(\pi - 4x)^2}.$$

$$12.5 \lim_{x \rightarrow 1} \frac{1 + \cos \pi x}{\operatorname{tg}^2 \pi x}.$$

$$12.6 \lim_{x \rightarrow \pi/2} \frac{\operatorname{tg} 3x}{\operatorname{tg} x}.$$

$$12.7 \lim_{x \rightarrow \pi} \frac{\sin^2 x - \operatorname{tg}^2 x}{(x - \pi)^4}.$$

$$12.8 \lim_{x \rightarrow 1} \frac{\sqrt{x^2 - x + 1} - 1}{\operatorname{tg} \pi x}.$$

$$12.9 \lim_{x \rightarrow \pi} \frac{\cos 5x - \cos 3x}{\sin^2 x}.$$

$$12.10 \lim_{x \rightarrow 2\pi} \frac{\sin 7x - \sin 3x}{e^{x^2} - e^{4\pi^2}}.$$

$$12.11 \lim_{x \rightarrow 2} \frac{\sin 7\pi x}{\sin 8\pi x}.$$

$$12.12 \lim_{x \rightarrow 2} \frac{\ln(5-2x)}{\sqrt{10-3x}-2}.$$

$$12.13 \lim_{x \rightarrow 1} \frac{\sqrt{x^2-3x+3}-1}{\sin \pi x}.$$

$$12.14 \lim_{x \rightarrow \pi} \frac{x^2 - \pi^2}{\sin x}.$$

$$12.15 \lim_{x \rightarrow 1} \frac{3^{5x-3} - 3^{2x^2}}{\operatorname{tg} \pi x}.$$

$$12.16 \lim_{x \rightarrow 4} \frac{2^x - 16}{\sin \pi x}.$$

$$12.17 \lim_{x \rightarrow \pi/2} \frac{\ln 2x - \ln \pi}{\sin(5x/2) \cos x}.$$

$$12.18 \lim_{x \rightarrow \pi/4} \frac{\ln \operatorname{tg} x}{\cos 2x}.$$

$$12.19 \lim_{x \rightarrow \pi} \frac{e^\pi - e^x}{\sin 5x - \sin 3x}.$$

$$12.20 \lim_{x \rightarrow 2} \frac{\ln(9-2x^2)}{\sin 2\pi x}.$$

$$12.21 \lim_{x \rightarrow 2} \frac{1-2^{4-x^2}}{2(\sqrt{2x}-\sqrt{3x^2-5x+2})}.$$

$$12.22 \lim_{x \rightarrow 1} \frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}.$$

$$12.23 \lim_{x \rightarrow -2} \frac{\operatorname{tg} \pi x}{x+2}.$$

$$12.24 \lim_{x \rightarrow \pi} \frac{1-\sin(x/2)}{\pi-x}.$$

$$12.25 \lim_{x \rightarrow \pi/3} \frac{1-2\cos x}{\pi-3x}.$$

$$12.26 \lim_{x \rightarrow 2} \frac{\operatorname{arctg}(x^2-2x)}{\sin 3\pi x}.$$

$$12.27 \lim_{x \rightarrow 1} \frac{1-x^2}{\sin \pi x}.$$

$$12.28 \lim_{x \rightarrow 1} \frac{\cos(\pi x/2)}{1-\sqrt{x}}.$$

$$12.29 \lim_{x \rightarrow 1} \frac{3-\sqrt{10-x}}{\sin 3\pi x}.$$

$$12.30 \lim_{x \rightarrow \pi} \frac{\sin 5x}{\operatorname{tg} 3x}.$$

$$12.31 \lim_{x \rightarrow \pi} \frac{\cos 3x - \cos x}{\operatorname{tg}^2 2x}.$$

Задача 13. Вычислить пределы функций.

$$13.1. \lim_{x \rightarrow \pi/2} \frac{2^{\cos^2 x} - 1}{\ln \sin x}.$$

$$13.2. \lim_{x \rightarrow 1/2} \frac{(2x-1)^2}{e^{\sin \pi x} - e^{-\sin 3\pi x}}.$$

$$13.3 \lim_{x \rightarrow 2} \frac{\ln(x - \sqrt[3]{2x-3})}{\sin(\pi x/2) - \sin[(x-1)\pi]}.$$

$$13.4. \lim_{x \rightarrow 2} \frac{\operatorname{tg} x - \operatorname{tg} 2}{\sin \ln(x-1)}.$$

$$13.5. \lim_{x \rightarrow \pi/2} \frac{e^{\operatorname{tg} 2x} - e^{-\sin 2x}}{\sin x - 1}.$$

$$13.7. \lim_{x \rightarrow 3} \frac{\sin(\sqrt{2x^2 - 3x - 5} - \sqrt{1+x})}{\ln(x-1) - \ln(x+1) + \ln 2}.$$

$$13.9. \lim_{x \rightarrow 1/2} \frac{\ln(4x-1)}{\sqrt{1-\cos \pi x} - 1}.$$

$$13.11. \lim_{x \rightarrow 3} \frac{2^{\sin \pi x} - 1}{\ln(x^3 - 6x - 8)}.$$

$$13.13. \lim_{x \rightarrow 2} \frac{\operatorname{tg} \ln(3x-5)}{e^{x+3} - e^{x^2+1}}.$$

$$13.15. \lim_{x \rightarrow 1} \frac{\sqrt[3]{1+\ln^2 x} - 1}{1 + \cos \pi x}.$$

$$13.17. \lim_{x \rightarrow 3} \frac{\ln(2x-5)}{e^{\sin \pi x} - 1}.$$

$$13.19. \lim_{x \rightarrow \pi/2} \frac{e^{\sin 2x} - e^{\operatorname{tg} 2x}}{\ln(2x/\pi)}.$$

$$13.21. \lim_{x \rightarrow 1} \frac{\sqrt{2^x + 7} - \sqrt{2^{x+1} + 5}}{x^3 - 1}.$$

$$13.23. \lim_{x \rightarrow \pi} \frac{(x^3 - \pi^3) \sin 5x}{e^{\sin^2 x} - 1}.$$

$$13.25. \lim_{x \rightarrow \pi} \frac{\ln \cos 2x}{\ln \cos 4x}.$$

$$13.27. \lim_{x \rightarrow a} \frac{a^{x^2-a^2} - 1}{\operatorname{tg} \ln(x/a)}.$$

$$13.29. \lim_{x \rightarrow a\pi} \frac{\ln(\cos(x/a) + 2)}{a^{a^2 \pi^2/x^2 - a\pi/x} - a^{a\pi/x-1}}.$$

$$13.6. \lim_{x \rightarrow \pi/6} \frac{\ln \sin 3x}{(6x - \pi)^2}.$$

$$13.8. \lim_{x \rightarrow 2\pi} \frac{(x - 2\pi)^2}{\operatorname{tg}(\cos x - 1)}.$$

$$13.10. \lim_{x \rightarrow -2} \frac{\arcsin(x+2)/2}{3\sqrt{2+x+x^2} - 9}.$$

$$13.12. \lim_{x \rightarrow \pi} \frac{\ln \cos 2x}{(1 - \pi/x)^2}.$$

$$13.14. \lim_{x \rightarrow 2\pi} \frac{\ln \cos x}{3^{\sin 2x} - 1}.$$

$$13.16. \lim_{x \rightarrow \pi} \frac{\cos(x/2)}{e^{\sin x} - e^{\sin 4x}}.$$

$$13.18. \lim_{x \rightarrow \pi/3} \frac{e^{\sin^2 6x} - e^{\sin^2 3x}}{\log_3 \cos 6x}.$$

$$13.20. \lim_{x \rightarrow -2} \frac{\operatorname{tg}(e^{x+2} - e^{x^2-4})}{\operatorname{tg} x + \operatorname{tg} 2}.$$

$$13.22. \lim_{x \rightarrow \pi} \frac{\ln(2 + \cos x)}{(3^{\sin x} - 1)^2}.$$

$$13.24. \lim_{x \rightarrow -1} \frac{\operatorname{tg}(x+1)}{e^{\sqrt[3]{x^3-4x^2+6}} - e}.$$

$$13.26. \lim_{x \rightarrow \pi/2} \frac{\ln \sin x}{(2x - \pi)^2}.$$

$$13.28. \lim_{x \rightarrow -3} \frac{\sin(e^{\sqrt[3]{1-x^2}/2} - e^{\sqrt[3]{x+2}})}{\operatorname{arctg}(x+3)}.$$

$$13.30. \lim_{x \rightarrow \pi} \frac{\operatorname{tg}(3^{\pi/x} - 3)}{3^{\cos(3x/2)} - 1}.$$

$$13.31 \lim_{x \rightarrow \pi} \frac{\sin(x^2/\pi)}{2^{\sqrt{\sin x + 1}} - 2}.$$

Задача 14. Вычислить пределы функций.

$$14.1. \lim_{x \rightarrow 0} \frac{7^{2x} - 5^{3x}}{2x - \operatorname{arctg} 3x}.$$

$$14.2. \lim_{x \rightarrow 0} \frac{e^{3x} - e^{-2x}}{2 \arcsin x - \sin x}.$$

$$14.3. \lim_{x \rightarrow 0} \frac{6^{2x} - 7^{-2x}}{\sin 3x - 2x}.$$

$$14.4. \lim_{x \rightarrow 0} \frac{e^{5x} - e^{3x}}{\sin 2x - \sin x}.$$

$$14.5. \lim_{x \rightarrow 0} \frac{3^{2x} - 5^{3x}}{\operatorname{arctg} x + x^3}.$$

$$14.6. \lim_{x \rightarrow 0} \frac{e^{2x} - e^{3x}}{\operatorname{arctg} x - x^2}.$$

$$14.7. \lim_{x \rightarrow 0} \frac{3^{5x} - 2^x}{x - \sin 9x}.$$

$$14.8. \lim_{x \rightarrow 0} \frac{e^{4x} - e^{-2x}}{2 \operatorname{arctg} x - \sin x}.$$

$$14.9. \lim_{x \rightarrow 0} \frac{12^x - 5^{-3x}}{2 \arcsin x - x}.$$

$$14.10. \lim_{x \rightarrow 0} \frac{e^{7x} - e^{-2x}}{\sin x - 2x}.$$

$$14.11. \lim_{x \rightarrow 0} \frac{3^{5x} - 2^{7x}}{\arcsin 2x - x}.$$

$$14.12. \lim_{x \rightarrow 0} \frac{e^{5x} - e^x}{\arcsin x + x^3}.$$

$$14.13. \lim_{x \rightarrow 0} \frac{4^x - 2^{7x}}{\operatorname{tg} 3x - x}.$$

$$14.14. \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\operatorname{tg} 2x - \sin x}.$$

$$14.15. \lim_{x \rightarrow 0} \frac{10^{2x} - 7^{-x}}{2 \operatorname{tg} x - \operatorname{arctg} x}.$$

$$14.16. \lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{\sin 3x - \sin 5x}.$$

$$14.17. \lim_{x \rightarrow 0} \frac{7^{3x} - 3^{2x}}{\operatorname{tg} x + x^3}.$$

$$14.18. \lim_{x \rightarrow 0} \frac{e^{4x} - e^{2x}}{2 \operatorname{tg} x - \sin x}.$$

$$17.19. \lim_{x \rightarrow 0} \frac{3^{2x} - 7^x}{\arcsin 3x - 5x}.$$

$$14.20. \lim_{x \rightarrow 0} \frac{e^{2x} - e^{-5x}}{2 \sin x - \operatorname{tg} x}.$$

$$14.21. \lim_{x \rightarrow 0} \frac{4^{5x} - 9^{-2x}}{\sin x - \operatorname{tg} x^3}.$$

$$14.22. \lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{\sin 3x - \operatorname{tg} 2x}.$$

$$14.23. \lim_{x \rightarrow 0} \frac{5^{2x} - 2^{3x}}{\sin x + \sin x^2}.$$

$$14.24. \lim_{x \rightarrow 0} \frac{e^x - e^{3x}}{\sin 3x - \operatorname{tg} 2x}.$$

$$14.25. \lim_{x \rightarrow 0} \frac{9^x - 2^{3x}}{\arctg 2x - 7x}.$$

$$14.27. \lim_{x \rightarrow 0} \frac{3^{5x} - 2^{-7x}}{2x - \operatorname{tg} x}.$$

$$14.29. \lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{x + \operatorname{tg} x^2}.$$

$$14.31. \lim_{x \rightarrow 0} \frac{2^{3x} - 3^{5x}}{\sin 7x - 2x}.$$

$$14.26. \lim_{x \rightarrow 0} \frac{e^x - e^{-2x}}{x + \sin x^2}.$$

$$14.28. \lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{\sin 2x - \sin x}.$$

$$14.30. \lim_{x \rightarrow 0} \frac{2^{3x} - 3^{2x}}{x + \arcsin x^3}.$$

Задача 15. Вычислить пределы функций.

$$15.1. \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{\sin^2 x}.$$

$$15.3. \lim_{x \rightarrow -1} \frac{x^3 + 1}{\sin(x+1)}.$$

$$15.5. \lim_{x \rightarrow 0} \frac{\sqrt{1 + \operatorname{tg} x} - \sqrt{1 + \sin x}}{x^3}.$$

$$15.7. \lim_{x \rightarrow 0} \frac{\sqrt{1 + x \sin x} - 1}{e^{x^2} - 1}.$$

$$15.9. \lim_{x \rightarrow \pi/3} \frac{1 - 2 \cos x}{\sin(\pi - 3x)}.$$

$$15.11. \lim_{x \rightarrow \pi/4} \frac{\sin x - \cos x}{\ln \operatorname{tg} x}.$$

$$15.13. \lim_{x \rightarrow 0} \frac{1 - \cos 2x + \operatorname{tg}^2 x}{x \sin 3x}.$$

$$15.15. \lim_{h \rightarrow 0} \frac{\ln(x+h) + \ln(x-h) - 2 \ln x}{h^2}, \quad x > 0.$$

$$15.17. \lim_{x \rightarrow 0} \frac{e^{\sin 2x} - e^{\sin x}}{\operatorname{tg} x}.$$

$$15.2. \lim_{x \rightarrow 0} \frac{1 + x \sin x - \cos 2x}{\sin^2 x}.$$

$$15.4. \lim_{x \rightarrow a} \frac{\operatorname{tg} x - \operatorname{tga}}{\ln x - \ln a}.$$

$$15.6. \lim_{x \rightarrow 0} \frac{e^{\alpha x} - e^{\beta x}}{\sin \alpha x - \sin \beta x}.$$

$$15.8. \lim_{x \rightarrow 0} \frac{x^2 (e^x - e^{-x})}{e^{x^3+1} - e}.$$

$$15.10. \lim_{x \rightarrow 1} \frac{1 - x^2}{\sin \pi x}.$$

$$15.12. \lim_{x \rightarrow b} \frac{a^x - a^b}{x - b}.$$

$$15.14. \lim_{x \rightarrow 0} \frac{\sin 2x - 2 \sin x}{x \ln \cos 5x}.$$

$$15.16. \lim_{x \rightarrow 1} \frac{1 - x}{\log_2 x}.$$

$$15.18. \lim_{x \rightarrow 1} \frac{2^x - 2}{\ln x}.$$

$$15.19. \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x-h)}{h}.$$

$$15.20. \lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{\sin 3x}.$$

$$15.21. \lim_{h \rightarrow 0} \frac{a^{x+h} + a^{x-h} - 2a^x}{h^2}.$$

$$15.22. \lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{1 - \cos \sqrt{x}}.$$

$$15.23. \lim_{x \rightarrow 3} \frac{\sqrt[3]{5+x} - 2}{\sin \pi x}.$$

$$15.24. \lim_{x \rightarrow \pi/6} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1}.$$

$$15.25. \lim_{x \rightarrow 10} \frac{\lg x - 1}{\sqrt{x-9} - 1}.$$

$$15.26. \lim_{x \rightarrow 0} \frac{3^{x+1} - 3}{\ln(1 + x\sqrt{1 + xe^x})}.$$

$$15.27. \lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - 1}{\sin^2 2x}.$$

$$15.28. \lim_{x \rightarrow 0} \frac{\sin bx - \sin ax}{\ln(\operatorname{tg}(\pi/4 + ax))}.$$

$$15.29. \lim_{x \rightarrow \pi/2} \frac{1 - \sin^3 x}{\cos^2 x}.$$

$$15.30. \lim_{x \rightarrow 3} \frac{\log_3 x - 1}{\operatorname{tg} \pi x}.$$

$$15.31. \lim_{x \rightarrow 1} \frac{e^x - e}{\sin(x^2 - 1)}.$$

Задача 16. Вычислить пределы функций.

$$16.1. \lim_{x \rightarrow 0} \left(1 - \ln(1 + x^3)\right)^{3/(x^2 \arcsin x)}.$$

$$16.2. \lim_{x \rightarrow 0} \left(\cos \sqrt{x}\right)^{1/x}.$$

$$16.3. \lim_{x \rightarrow 0} \left(\frac{1 + x \cdot 2^x}{1 + x \cdot 3^x}\right)^{1/x^2}.$$

$$16.4. \lim_{x \rightarrow 0} \left(2 - 3^{\operatorname{arctg}^2 \sqrt{x}}\right)^{2/\sin x}.$$

$$16.5. \lim_{x \rightarrow 0} \left(\frac{1 + \sin x \cos \alpha x}{1 + \sin x \cos \beta x}\right)^{\operatorname{ctg}^3 x}.$$

$$16.6. \lim_{x \rightarrow 0} \left(5 - \frac{4}{\cos x}\right)^{1/\sin^2 3x}.$$

$$16.7. \lim_{x \rightarrow 0} \left(1 - \ln(1 + \sqrt[3]{x})\right)^{x/\sin^4 \sqrt[3]{x}}.$$

$$16.8. \lim_{x \rightarrow 0} \left(2 - e^{\arcsin^2 \sqrt{x}}\right)^{3/x}.$$

$$16.9. \lim_{x \rightarrow 0} (\cos \pi x)^{1/(x \sin \pi x)}.$$

$$16.10. \lim_{x \rightarrow 0} \left(1 + \sin^2 3x\right)^{1/\ln \cos x}.$$

$$16.11. \lim_{x \rightarrow 0} \left(\operatorname{tg}\left(\frac{\pi}{4} - x\right)\right)^{\operatorname{ctg} x}.$$

$$16.12. \lim_{x \rightarrow 0} \left(1 - x \sin^2 x\right)^{1/\ln(1 + \pi x^3)}.$$

$$16.13. \lim_{x \rightarrow 0} \left(2 - 5^{\arcsin x^3}\right)^{(\operatorname{cosec}^2 x)/x}.$$

$$16.14. \lim_{x \rightarrow 0} (2 - \cos 3x)^{1/\ln(1+x^2)}.$$

$$16.15. \lim_{x \rightarrow 0} (2 - e^{\sin x})^{\operatorname{ctg} \pi x}.$$

$$16.16. \lim_{x \rightarrow 0} (\cos x)^{1/\ln(1+\sin^2 x)}.$$

$$16.17. \lim_{x \rightarrow 0} (2 - e^{x^2})^{1/\ln(1+\operatorname{tg}^2(\pi x/3))}.$$

$$16.18. \lim_{x \rightarrow 0} (3 - 2 \cos x)^{-\operatorname{cosec}^2 x}.$$

$$16.19. \lim_{x \rightarrow 0} (2 - 3^{\sin^2 x})^{1/\ln \cos x}.$$

$$16.20. \lim_{x \rightarrow 0} \sqrt{x^2 - \cos x}.$$

$$16.21. \lim_{x \rightarrow 0} \left(6 - \frac{5}{\cos x}\right)^{\operatorname{ctg}^2 x}.$$

$$16.22. \lim_{x \rightarrow 0} \left(3 - \frac{2}{\cos x}\right)^{\operatorname{cosec}^2 x}.$$

$$16.23. \lim_{x \rightarrow 0} \left(\frac{1 + \sin x \cos 2x}{1 + \sin x \cos 3x}\right)^{1/\sin x^3}.$$

$$16.24. \lim_{x \rightarrow 0} (2 - e^{x^2})^{1/(1-\cos \pi x)}.$$

$$16.25. \lim_{x \rightarrow 0} \left(1 + \ln \frac{1}{3} \operatorname{arctg}^6 \sqrt{x}\right)^{1/x^3}.$$

$$16.26. \lim_{x \rightarrow 0} \left(\frac{1 + \operatorname{tg} x \cos 2x}{1 + \operatorname{tg} x \cos 5x}\right)^{1/x^3}.$$

$$16.27. \lim_{x \rightarrow 0} \left(\frac{1 + x \cdot 3^x}{1 + x \cdot 7^x}\right)^{1/\operatorname{tg}^2 x}.$$

$$16.28. \lim_{x \rightarrow 0} (1 + \operatorname{tg}^2 x)^{1/\ln(1+3x^2)}.$$

$$16.29. \lim_{x \rightarrow 0} (1 - \ln \cos x)^{1/\operatorname{tg}^2 x}.$$

$$16.30. \lim_{x \rightarrow 0} \left(1 - \sin^2 \frac{x}{2}\right)^{1/\ln(1+\operatorname{tg}^2 3x)}.$$

$$16.31. \lim_{x \rightarrow 0} \left(\frac{1 + x^2 \cdot 2^x}{1 + x^2 \cdot 5^x}\right)^{1/\sin^3 x}.$$

Задача 17. Вычислить пределы функций.

$$17.1. \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x}\right)^{1+x}.$$

$$17.2. \lim_{x \rightarrow 0} \left(\frac{2+x}{3-x}\right)^x.$$

$$17.3. \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{x}\right)^{2/(x+2)}.$$

$$17.4. \lim_{x \rightarrow 0} \left(\frac{e^{3x} - 1}{x}\right)^{\cos^2(\pi/4+x)}.$$

$$17.5. \lim_{x \rightarrow 0} (\cos x)^{x+3}.$$

$$17.6. \lim_{x \rightarrow 0} \left(\frac{x^2 + 4}{x + 2} \right)^{x^2+3}.$$

$$17.7. \lim_{x \rightarrow 0} \left(\frac{\ln(1+x)}{6x} \right)^{x/(x+2)}.$$

$$17.8. \lim_{x \rightarrow 0} \left(\frac{\operatorname{tg} 4x}{x} \right)^{2+x}.$$

$$17.9. \lim_{x \rightarrow 0} \left(\frac{e^{x^3} - 1}{x^2} \right)^{(8x+3)/(1+x)}.$$

$$17.10. \lim_{x \rightarrow 0} \left(\frac{x+2}{x+4} \right)^{\cos x}.$$

$$17.11. \lim_{x \rightarrow 0} \left(\frac{\sin 6x}{2x} \right)^{2+x}.$$

$$17.12. \lim_{x \rightarrow 0} \left(\frac{e^{x^2} - 1}{x^2} \right)^{6/(1+x)}.$$

$$17.13. \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{\sin 3x} \right)^{x^2}.$$

$$17.14. \lim_{x \rightarrow 0} \left(\operatorname{tg} \left(x + \frac{\pi}{3} \right) \right)^{x+2}.$$

$$17.15. \lim_{x \rightarrow 0} \left(\frac{x^3 + 8}{3x^2 + 10} \right)^{x+2}.$$

$$17.16. \lim_{x \rightarrow 0} (\sin(x+2))^{3/(3+x)}.$$

$$17.17. \lim_{x \rightarrow 0} \left(\frac{2^{2x} - 1}{x} \right)^{x+1}.$$

$$17.18. \lim_{x \rightarrow 0} \left(\frac{x^4 + 5}{x + 10} \right)^{4/(x+2)}.$$

$$17.19. \lim_{x \rightarrow 0} \left(\frac{11x + 8}{12x + 1} \right)^{\cos^2 x}.$$

$$17.20. \lim_{x \rightarrow 0} \left(\frac{x^3 + 1}{x^3 + 8} \right)^{2/(x+1)}.$$

$$17.21. \lim_{x \rightarrow 0} \left(\frac{\ln(1+x^2)}{x^2} \right)^{3/(x+8)}.$$

$$17.22. \lim_{x \rightarrow 0} \left(\cos \frac{x}{\pi} \right)^{1+x}.$$

$$17.23. \lim_{x \rightarrow 0} \left(\frac{\arcsin x}{x} \right)^{2/(x+5)}.$$

$$17.24. \lim_{x \rightarrow 0} \left(\frac{\operatorname{arc} \operatorname{tg} 3x}{x} \right)^{x+2}.$$

$$17.25. \lim_{x \rightarrow 0} (e^x + x)^{\cos x^4}.$$

$$17.26. \lim_{x \rightarrow 0} \left(\frac{\sin 5x^2}{\sin x} \right)^{1/(x+6)}.$$

$$17.27. \lim_{x \rightarrow 0} \left(\operatorname{tg} \left(\frac{\pi}{4} - x \right) \right)^{(e^x - 1)/x}.$$

$$17.28. \lim_{x \rightarrow 0} \left(6 - \frac{5}{\cos x} \right)^{\operatorname{tg}^2 x}.$$

$$17.29. \lim_{x \rightarrow 0} \left(\frac{1+8x}{2+11x} \right)^{1/(x^2+1)} .$$

$$17.30. \lim_{x \rightarrow 0} \left(\frac{\arcsin^2 x}{\arcsin^2 4x} \right)^{2x+1} .$$

$$17.31. \lim_{x \rightarrow 0} \left(\frac{x^3+4}{x^3+9} \right)^{1/(x+2)} .$$

Задача 18. Вычислить пределы функций.

$$18.1. \lim_{x \rightarrow 1} \left(\frac{3x-1}{x+1} \right)^{1/(\sqrt[3]{x}-1)} .$$

$$18.2. \lim_{x \rightarrow a} \left(\frac{\sin x}{\sin a} \right)^{1/(x-a)} .$$

$$18.3. \lim_{x \rightarrow 1} \left(\frac{2x-1}{x} \right)^{1/(\sqrt[3]{x}-1)} .$$

$$18.4. \lim_{x \rightarrow 2} \left(\frac{\cos x}{\cos 2} \right)^{1/(x-2)} .$$

$$18.5. \lim_{x \rightarrow 8} \left(\frac{2x-7}{x+1} \right)^{1/(\sqrt[3]{x}-2)} .$$

$$18.6. \lim_{x \rightarrow \pi/4} (\operatorname{tg} x)^{1/\cos(3\pi/4-x)} .$$

$$18.7. \lim_{x \rightarrow 1} \left(\frac{2x-1}{x} \right)^{1/(\sqrt[5]{x}-1)} .$$

$$18.8. \lim_{x \rightarrow a} \left(2 - \frac{x}{a} \right)^{\operatorname{tg} \frac{\pi x}{2a}} .$$

$$18.9. \lim_{x \rightarrow 2\pi} (\cos x)^{\operatorname{ctg} 2x/\sin 3x} .$$

$$18.10. \lim_{x \rightarrow 2\pi} (\cos x)^{1/\sin^2 2x} .$$

$$18.11. \lim_{x \rightarrow 3} \left(\frac{6-x}{3} \right)^{\operatorname{tg} \frac{\pi x}{6}} .$$

$$18.12. \lim_{x \rightarrow 4\pi} (\cos x)^{\operatorname{ctg} x/\sin 4x} .$$

$$18.13. \lim_{x \rightarrow 1} (3-2x)^{\operatorname{tg} \frac{\pi x}{2}} .$$

$$18.14. \lim_{x \rightarrow 4\pi} (\cos x)^{\frac{5}{\operatorname{tg} 5x \sin 2x}} .$$

$$18.15. \lim_{x \rightarrow 3} \left(\frac{9-2x}{3} \right)^{\operatorname{tg} \frac{\pi x}{6}} .$$

$$18.16. \lim_{x \rightarrow \pi/2} (\sin x)^{6\operatorname{tg} x \operatorname{tg} 3x} .$$

$$18.17. \lim_{x \rightarrow 1} (2e^{x-1} - 1)^{x/(x-1)} .$$

$$18.18. \lim_{x \rightarrow \pi/2} \left(\operatorname{tg} \frac{x}{2} \right)^{1/(x-\pi/2)} .$$

$$18.19. \lim_{x \rightarrow 1} \left(2e^{x-1} - 1 \right)^{(3x-1)/(x-1)} .$$

$$18.20. \lim_{x \rightarrow \pi/2} (1 + \cos 3x)^{\sec x} .$$

$$18.21. \lim_{x \rightarrow 2} \left(2e^{x-2} - 1 \right)^{(3x+2)/(x-2)} .$$

$$18.22. \lim_{x \rightarrow 1} \left(\frac{\sin(x-1)}{x-1} \right)^{\frac{\sin(x-1)}{x-1-\sin(x-1)}} .$$

$$18.23. \lim_{x \rightarrow 1} \left(\frac{2-x}{x} \right)^{1/\ln(2-x)} .$$

$$18.24. \lim_{x \rightarrow \pi/2} \left(\operatorname{ctg} \frac{x}{2} \right)^{1/\cos x} .$$

$$18.25. \lim_{x \rightarrow 1} (2-x)^{\frac{\sin(\pi x/2)}{\ln(2-x)}} .$$

$$18.26. \lim_{x \rightarrow 3} \left(\frac{\sin x}{\sin 3} \right)^{1/(x-3)} .$$

$$18.27. \lim_{x \rightarrow 1} \left(\frac{x+1}{2x} \right)^{\frac{\ln(x+2)}{\ln(2-x)}} .$$

$$18.28. \lim_{x \rightarrow \pi/2} (\sin x)^{\frac{18 \sin x}{\operatorname{ctg} x}} .$$

$$18.29. \lim_{x \rightarrow 1} \left(\frac{1}{x} \right)^{\frac{\ln(x+1)}{\ln(2-x)}} .$$

$$18.30. \lim_{x \rightarrow \pi} \left(\operatorname{ctg} \frac{x}{4} \right)^{1/\cos(x/2)} .$$

$$18.31. \lim_{x \rightarrow 1} \left(\frac{2x-1}{x} \right)^{\frac{\ln(3+2x)}{\ln(2-x)}} .$$

Задача 19. Вычислить пределы функций.

$$19.1. \lim_{x \rightarrow e} \left(\frac{\ln x - 1}{x - e} \right)^{\sin \frac{\pi}{2e} x} .$$

$$19.2. \lim_{x \rightarrow \pi/4} (\operatorname{tg} x)^{\operatorname{ctg} x} .$$

$$19.3. \lim_{x \rightarrow \pi/4} \left(\frac{\ln \operatorname{tg} x}{1 - \operatorname{ctg} x} \right)^{1/(x+\pi/4)} .$$

$$19.4. \lim_{x \rightarrow 2} (\sin x)^{3/(1+x)} .$$

$$19.5. \lim_{x \rightarrow 2} \left(\frac{\sin 3\pi x}{\sin \pi x} \right)^{\sin^2(x-2)} .$$

$$19.6. \lim_{x \rightarrow \pi/6} (\sin x)^{6x/\pi} .$$

$$19.7. \lim_{x \rightarrow 3} \left(2 - \frac{x}{3} \right)^{\sin \pi x} .$$

$$19.8. \lim_{x \rightarrow 1} \left(\frac{1+x}{2+x} \right)^{(1-x^2)/(1-x)} .$$

$$19.9. \lim_{x \rightarrow 1} \left(1 + e^x \right)^{\frac{\sin \pi x}{1-x}} .$$

$$19.10. \lim_{x \rightarrow 1} \left(\frac{\operatorname{tg} 9\pi x}{\sin 4\pi x} \right)^{x/(x+1)} .$$

$$19.11. \lim_{x \rightarrow 3} \left(\frac{\arcsin(x-3)}{\sin 3\pi x} \right)^{x^2-8}.$$

$$19.12. \lim_{x \rightarrow \pi/4} (\sin 2x)^{\frac{x^2-\pi^2/16}{x-\pi/4}}.$$

$$19.13. \lim_{x \rightarrow 1} \left(\operatorname{arctg} \frac{x-3/4}{(x-1)^2} \right)^{x+1}.$$

$$19.14. \lim_{x \rightarrow \pi} \left(\operatorname{ctg} \frac{x}{4} \right)^{\sin(x-\pi)}.$$

$$19.15. \lim_{x \rightarrow a} \left(\frac{\sin x - \sin a}{x-a} \right)^{x^2/a^2}.$$

$$19.16. \lim_{x \rightarrow 2} \left(\frac{\sqrt{x+2}-2}{x^2-4} \right)^{1/x}.$$

$$19.17. \lim_{x \rightarrow \pi/4} (\sin x + \cos x)^{1/\operatorname{tg} x}.$$

$$19.18. \lim_{x \rightarrow \pi/8} (\operatorname{tg} 2x)^{\sin(\pi/8+x)}.$$

$$19.19. \lim_{x \rightarrow 1} (\arcsin x)^{\operatorname{tg} \pi x}.$$

$$19.20. \lim_{x \rightarrow \pi} (x + \sin x)^{\sin x + x}.$$

$$19.21. \lim_{x \rightarrow 1} (\ln^2 ex)^{1/(x^2+1)}.$$

$$19.22. \lim_{x \rightarrow 1} (\sqrt{x} + 1)^{\pi/\operatorname{arctg} x}.$$

$$19.23. \lim_{x \rightarrow 1} \left(\frac{x^3-1}{x-1} \right)^{1/x^2}.$$

$$19.24. \lim_{x \rightarrow 1} \left(\frac{e^{\sin \pi x} - 1}{x-1} \right)^{x^2+1}.$$

$$19.25. \lim_{x \rightarrow 2} (\cos \pi x)^{\operatorname{tg}(x-2)}.$$

$$19.26. \lim_{x \rightarrow 1/2} (\arcsin x + \arccos x)^{1/x}.$$

$$19.27. \lim_{x \rightarrow \pi/2} (\cos x + 1)^{\sin x}.$$

$$19.28. \lim_{x \rightarrow 1} (\sqrt[3]{x} + x - 1)^{\sin(\pi x/4)}.$$

$$19.29. \lim_{x \rightarrow 1} \left(\frac{x^2+2x-3}{x^2+4x-5} \right)^{1/(2-x)}.$$

$$19.30. \lim_{x \rightarrow 1} \left(\frac{1+\cos \pi x}{\operatorname{tg}^2 \pi x} \right)^{x^2}.$$

$$19.31. \lim_{x \rightarrow 1} \left(\frac{e^{2x} - e^2}{x-1} \right)^{x+1}.$$

Задача 20. Вычислить предел функции или числовой последовательности.

$$20.1. \lim_{x \rightarrow 0} \sqrt{4 \cos 3x + x \operatorname{arctg}(1/x)}.$$

$$20.2. \lim_{x \rightarrow \pi/2} \sqrt{3 \sin x + (2x - \pi) \sin \frac{x}{2x - \pi}}.$$

$$20.3. \lim_{n \rightarrow \infty} \frac{2n - \sin n}{\sqrt{n} - \sqrt[3]{n^3 - 7}}.$$

$$20.4. \lim_{x \rightarrow 0} \frac{\operatorname{tg} x \cos(1/x) + \lg(2+x)}{\lg(4+x)}.$$

$$20.5. \lim_{n \rightarrow \infty} \frac{e^{1/n} + \sin \frac{n}{n^2 + 1} \cdot \cos n}{1 + \cos(1/n)}.$$

$$20.6. \lim_{n \rightarrow \infty} \frac{\sqrt[4]{2 + n^5} - \sqrt{2n^3 + 3}}{(n + \sin n)\sqrt{7n}}.$$

$$20.7. \lim_{x \rightarrow \pi/4} \frac{\sqrt[3]{\operatorname{tg} x} + (4x - \pi) \cos \frac{x}{4x - \pi}}{\lg(2 + \operatorname{tg} x)}.$$

$$20.8. \lim_{n \rightarrow \infty} \left(\sin \sqrt{n^2 + 1} \cdot \operatorname{arctg} \frac{n}{n^2 + 1} \right).$$

$$20.9. \lim_{n \rightarrow \infty} \frac{n^2 - \sqrt{3n^5 - 7}}{(n^2 - n \cos n + 1)\sqrt{n}}.$$

$$20.10. \lim_{n \rightarrow \infty} \frac{3 \sin n + \sqrt{n-1}}{n + \sqrt{n+1}}.$$

$$20.11. \lim_{n \rightarrow \infty} \frac{(1 - \cos n) \sqrt[3]{n}}{\sqrt{2n+1} - 1}.$$

$$20.12. \lim_{x \rightarrow 0} \ln \left(2 + \sqrt{\operatorname{arctg} x \cdot \sin \frac{1}{x}} \right).$$

$$20.13. \lim_{x \rightarrow -2} \sqrt{\frac{1 + \cos \pi x}{4 + (x+2) \sin \frac{x}{x+2}}}.$$

$$20.14. \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{n^4 - 3} + \sin n}.$$

$$20.15. \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 + \cos n} + \sqrt{3n^2 + 2}}{\sqrt[5]{n^6 + 1}}.$$

$$20.16. \lim_{x \rightarrow 0} \frac{\sqrt[3]{\operatorname{tg} x \operatorname{arctg} \frac{1}{x}} + 3}{2 - \lg(1 + \sin x)}.$$

$$20.17. \lim_{x \rightarrow 0} \sqrt{\operatorname{arctg} x \cdot \sin^2 \frac{1}{x} + 5 \cos x}.$$

$$20.18. \lim_{x \rightarrow 0} \sqrt{4 \cos x + \sin \frac{1}{x} \cdot \ln(1+x)}.$$

$$20.19. \lim_{x \rightarrow 0} \sqrt{2 \cos^2 x + (e^x - 1) \sin \frac{1}{x}}.$$

$$20.20. \lim_{x \rightarrow 0} \frac{2 + \ln \left(e + x \sin \frac{1}{x} \right)}{\cos x + \sin x}.$$

$$20.21. \lim_{x \rightarrow 0} \ln \left[\left(e^{x^2} - \cos x \right) \cos \frac{1}{x} + \operatorname{tg} \left(x + \frac{\pi}{3} \right) \right].$$

$$20.22. \lim_{x \rightarrow 0} \frac{\cos x + \ln(1+x) \sqrt{2 + \cos \frac{1}{x}}}{2 + e^x}.$$

$$20.23. \lim_{x \rightarrow 1} \frac{\cos 2\pi x}{2 + (e^{\sqrt{x-1}} - 1) \operatorname{arctg} \frac{x+2}{x-1}}.$$

$$20.24. \lim_{x \rightarrow 0} \sqrt{(e^{\sin x} - 1) \cos \frac{1}{x} + 4 \cos x}.$$

$$20.25. \lim_{x \rightarrow 0} \frac{\cos(1+x)}{\left(2 + \sin \frac{1}{x} \right) \ln(1+x) + 2}.$$

$$20.26. \lim_{x \rightarrow 2} \sqrt[3]{\lg(x+2) + \sin \sqrt{4-x^2}} \cos \frac{x+2}{x-2}.$$

$$20.27. \lim_{x \rightarrow \pi/2} \frac{2 + \cos x \sin \frac{2}{2x-\pi}}{3 + 2x \sin x}.$$

$$20.28. \lim_{x \rightarrow 1} \operatorname{tg} \left(\cos x + \sin \frac{x-1}{x+1} \cos \frac{x+1}{x-1} \right).$$

$$20.29. \lim_{x \rightarrow 0} \sqrt{x \left(2 + \sin \frac{1}{x} \right) + 4 \cos x}.$$

$$20.30. \lim_{x \rightarrow 1} \frac{\sin x + \sin \pi x \cdot \operatorname{arctg} \frac{1+x}{1-x}}{1 + \cos x}.$$

$$20.31. \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 3n - 1} + \sqrt[3]{2n^2 + 1}}{n + 2 \sin n}.$$